Derivation of Tables 3 through 5 in Okada (1992)

[I] Derivation of Table 3 (x-Derivative)

Table 3 can be derived by differentiation of Table 2 with x-coordinate (refer Appendix : Table of Differentiation). In the following, the notation is matched with Tables in Okada (1992).

Displacement : $u^0(x,y,z)=\frac{M_0}{2\pi\mu}\left[{u^0}_A(x,y,z)-{u^0}_A(x,y,-z)+{u^0}_B(x,y,z)+z{u^0}_C(x,y,z)\right]$

x-Derivative : $\frac{\partial u^0}{\partial x}(x,y,z)=\frac{M_0}{2\pi\mu}\left[\frac{\partial {u^0}_A}{\partial x}(x,y,z)-\frac{\partial {u^0}_A}{\partial x}(x,y,-z)+\frac{\partial {u^0}_B}{\partial x}(x,y,z)+z\frac{\partial {u^0}_C}{\partial x}(x,y,z)\right]$

(1) Strike slip

$$u_A^0=\begin{pmatrix}\frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3x^2q}{R^5}\\ \frac{1-\alpha}{2}\frac{x}{R^3}\sin\delta+\frac{\alpha}{2}\frac{3xyq}{R^5}\\ -\frac{1-\alpha}{2}\frac{x}{R^3}\cos\delta+\frac{\alpha}{2}\frac{3xdq}{R^5}\end{pmatrix}\quad u_B^0=\begin{pmatrix}-\frac{3x^2q}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin\delta\\ -\frac{3xyq}{R^5}-\frac{1-\alpha}{\alpha}I_2^0\sin\delta\\ -\frac{3cxq}{R^5}-\frac{1-\alpha}{\alpha}I_4^0\sin\delta\end{pmatrix}\quad u_C^0=\begin{pmatrix}-(1-\alpha)\frac{A_3}{R^3}\cos\delta+\alpha\frac{3cq}{R^5}A_5\\ (1-\alpha)\frac{3xy}{R^5}\cos\delta+\alpha\frac{3cx}{R^5}\left(\sin\delta-\frac{5yq}{R^2}\right)\\ -(1-\alpha)\frac{3xy}{R^5}\sin\delta+\alpha\frac{3cx}{R^5}\left(\cos\delta+\frac{5dq}{R^2}\right)\end{pmatrix}$$

where, $d=c-z$, $q=y\sin\delta-d\cos\delta$, $R^2=x^2+y^2+d^2$ and

$$I_1^0=y\left[\frac{1}{R(R+d)^2}-x^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_2^0=x\left[\frac{1}{R(R+d)^2}-y^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_4^0=-xy\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial x}=\begin{pmatrix}-\frac{1-\alpha}{2}\frac{3xq}{R^5}+\frac{\alpha}{2}\frac{3xq}{R^5}(1+A_5)\\ \frac{1-\alpha}{2}\frac{A_3}{R^3}\sin\delta+\frac{\alpha}{2}\frac{3yq}{R^5}A_5\\ -\frac{1-\alpha}{2}\frac{A_3}{R^3}\cos\delta+\frac{\alpha}{2}\frac{3dq}{R^5}A_5\end{pmatrix}\quad \frac{\partial u_B^0}{\partial x}=\begin{pmatrix}-\frac{3xq}{R^5}(1+A_5)-\frac{1-\alpha}{\alpha}J_1^0\sin\delta\\ -\frac{3yq}{R^5}A_5-\frac{1-\alpha}{\alpha}J_2^0\sin\delta\\ -\frac{3cq}{R^5}A_5-\frac{1-\alpha}{\alpha}K_1^0\sin\delta\end{pmatrix}\quad \begin{aligned}A_3&=1-\frac{3x^2}{R^2}\\ A_5&=1-\frac{5x^2}{R^2}\\ A_7&=1-\frac{7x^2}{R^2}\end{aligned}$$

$$\frac{\partial u_C^0}{\partial x}=\begin{pmatrix}(1-\alpha)\frac{3x}{R^5}(2+A_5)\cos\delta-\alpha\frac{15cxq}{R^7}(2+A_7)\\ (1-\alpha)\frac{3y}{R^5}A_5\cos\delta+\alpha\frac{3c}{R^5}\left(A_5\sin\delta-\frac{5yq}{R^2}A_7\right)\\ -(1-\alpha)\frac{3y}{R^5}A_5\sin\delta+\alpha\frac{3c}{R^5}\left(A_5\cos\delta+\frac{5dq}{R^2}A_7\right)\end{pmatrix}$$

$$\begin{aligned}J_1^0&\equiv\frac{\partial}{\partial x}I_1^0=-3xy\left[\frac{3R+d}{R^3(R+d)^3}-x^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}\right]\\ J_2^0&\equiv\frac{\partial}{\partial x}I_2^0=\frac{1}{R^3}-\frac{3}{R(R+d)^2}+3x^2y^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}\\ K_1^0&\equiv\frac{\partial}{\partial x}I_4^0=-y\left[\frac{2R+d}{R^3(R+d)^2}-x^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]\end{aligned}$$

The above three vectors correspond to the contents of the row of Strike-Slip in Table 3.

(2) Dip slip

$$u_A^0=\begin{pmatrix}\frac{\alpha}{2}\frac{3xpq}{R^5}\\ \frac{1-\alpha}{2}\frac{s}{R^3}+\frac{\alpha}{2}\frac{3ypq}{R^5}\\ -\frac{1-\alpha}{2}\frac{t}{R^3}+\frac{\alpha}{2}\frac{3dpq}{R^5}\end{pmatrix}\quad u_B^0=\begin{pmatrix}-\frac{3xpq}{R^5}+\frac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta\\ -\frac{3ypq}{R^5}+\frac{1-\alpha}{\alpha}I_1^0\sin\delta\cos\delta\\ -\frac{3cpq}{R^5}+\frac{1-\alpha}{\alpha}I_5^0\sin\delta\cos\delta\end{pmatrix}\quad u_C^0=\begin{pmatrix}(1-\alpha)\frac{3xt}{R^5}-\alpha\frac{15cxpq}{R^7}\\ (1-\alpha)\left(\frac{3yt}{R^5}-\frac{\cos 2\delta}{R^3}\right)-\alpha\frac{15cypq}{R^7}+\alpha\frac{3cs}{R^5}\\ -(1-\alpha)\frac{A_3}{R^3}\sin\delta\cos\delta+\alpha\frac{15cdpq}{R^7}+\alpha\frac{3ct}{R^5}\end{pmatrix}$$

where, $d=c-z$, $\begin{cases}p=y\cos\delta+d\sin\delta\\ q=y\sin\delta-d\cos\delta\end{cases}$, $\begin{cases}s=p\sin\delta+q\cos\delta=y\sin 2\delta-d\cos 2\delta\\ t=p\cos\delta-q\sin\delta=y\cos 2\delta+d\sin 2\delta\end{cases}$ and

$$I_1^0=y\left[\frac{1}{R(R+d)^2}-x^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_2^0=x\left[\frac{1}{R(R+d)^2}-y^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_3^0=\frac{x}{R^3}-I_2^0\quad I_5^0=\frac{1}{R(R+d)}-x^2\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial x}=\begin{pmatrix}\frac{\alpha}{2}\frac{3pq}{R^5}A_5\\ -\frac{1-\alpha}{2}\frac{3xs}{R^5}-\frac{\alpha}{2}\frac{15xypq}{R^7}\\ \frac{1-\alpha}{2}\frac{3xt}{R^5}-\frac{\alpha}{2}\frac{15xdpq}{R^7}\end{pmatrix}\quad \frac{\partial u_B^0}{\partial x}=\begin{pmatrix}-\frac{3pq}{R^5}A_5+\frac{1-\alpha}{\alpha}J_3^0\sin\delta\cos\delta\\ \frac{15xypq}{R^7}+\frac{1-\alpha}{\alpha}J_1^0\sin\delta\cos\delta\\ \frac{15cxpq}{R^7}+\frac{1-\alpha}{\alpha}K_3^0\sin\delta\cos\delta\end{pmatrix}$$

$$\frac{\partial u_C^o}{\partial x}=\begin{pmatrix}(1-\alpha)\dfrac{3t}{R^5}A_5 & -\alpha\dfrac{15cpq}{R^7}A_7\\ (1-\alpha)\dfrac{3x}{R^5}\left(\cos2\delta-\dfrac{5yt}{R^2}\right) & -\alpha\dfrac{15cx}{R^7}\left(s-\dfrac{7ypq}{R^2}\right)\\ (1-\alpha)\dfrac{3x}{R^5}(2+A_5)\sin\delta\cos\delta & -\alpha\dfrac{15cx}{R^7}\left(t+\dfrac{7dpq}{R^2}\right)\end{pmatrix}$$

$$J_1^0\equiv\frac{\partial}{\partial x}I_1^0=-3xy\left[\frac{3R+d}{R^3(R+d)^3}-x^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}\right]$$
$$J_2^0\equiv\frac{\partial}{\partial x}I_2^0=\frac{1}{R^3}-\frac{3}{R(R+d)^2}+3x^2y^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}$$
$$J_3^0\equiv\frac{\partial}{\partial x}I_3^0=\frac{A_3}{R^3}-J_2^0$$
$$K_3^0\equiv\frac{\partial}{\partial x}I_5^0=-3x\frac{2R+d}{R^3(R+d)^2}+x^3\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}$$

The above three vectors correspond to the contents of the row of Dip-Slip in Table 3.

(3) Tensile

$$u_A^o=\begin{pmatrix}\dfrac{1-\alpha}{2}\dfrac{x}{R^3}-\dfrac{\alpha}{2}\dfrac{3xq^2}{R^5}\\ \dfrac{1-\alpha}{2}\dfrac{t}{R^3}-\dfrac{\alpha}{2}\dfrac{3yq^2}{R^5}\\ \dfrac{1-\alpha}{2}\dfrac{s}{R^3}-\dfrac{\alpha}{2}\dfrac{3dq^2}{R^5}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\dfrac{3xq^2}{R^5}-\dfrac{1-\alpha}{\alpha}I_3^0\sin^2\delta\\ \dfrac{3yq^2}{R^5}-\dfrac{1-\alpha}{\alpha}I_1^0\sin^2\delta\\ \dfrac{3cq^2}{R^5}-\dfrac{1-\alpha}{\alpha}I_5^0\sin^2\delta\end{pmatrix}$$

$$u_C^o=\begin{pmatrix}-(1-\alpha)\dfrac{3xs}{R^5} & +\alpha\dfrac{15cxq^2}{R^7}-\alpha\dfrac{3xz}{R^5}\\ (1-\alpha)\left(\dfrac{\sin2\delta}{R^3}-\dfrac{3ys}{R^5}\right) & +3c\alpha\dfrac{t-y}{R^5}+\alpha\dfrac{15cyq^2}{R^7}-\alpha\dfrac{3yz}{R^5}\\ -(1-\alpha)\left(\dfrac{\cos^2\delta}{R^3}+\dfrac{3x^2}{R^5}\sin^2\delta\right) & -3c\alpha\dfrac{s-d}{R^5}-\alpha\dfrac{15cdq^2}{R^7}+\alpha\dfrac{3dz}{R^5}\end{pmatrix}$$

$$\frac{\partial u_A^o}{\partial x}=\begin{pmatrix}\dfrac{1-\alpha}{2}\dfrac{A_3}{R^3}-\dfrac{\alpha}{2}\dfrac{3q^2}{R^5}A_5\\ -\dfrac{1-\alpha}{2}\dfrac{3xt}{R^5}+\dfrac{\alpha}{2}\dfrac{15xyq^2}{R^7}\\ -\dfrac{1-\alpha}{2}\dfrac{3xs}{R^5}+\dfrac{\alpha}{2}\dfrac{15xdq^2}{R^7}\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial x}=\begin{pmatrix}\dfrac{3q^2}{R^5}A_5-\dfrac{1-\alpha}{\alpha}J_3^0\sin^2\delta\\ -\dfrac{15xyq^2}{R^7}-\dfrac{1-\alpha}{\alpha}J_1^0\sin^2\delta\\ -\dfrac{15cxq^2}{R^7}-\dfrac{1-\alpha}{\alpha}K_3^0\sin^2\delta\end{pmatrix}\qquad \begin{aligned}A_3&=1-\frac{3x^2}{R^2}\\ A_5&=1-\frac{5x^2}{R^2}\\ A_7&=1-\frac{7x^2}{R^2}\end{aligned}$$

$$\frac{\partial u_C^o}{\partial x}=\begin{pmatrix}-(1-\alpha)\dfrac{3s}{R^5}A_5 & +\alpha\dfrac{15cq^2}{R^7}A_7 & -\alpha\dfrac{3z}{R^5}A_5\\ -(1-\alpha)\dfrac{3x}{R^5}\left(\sin2\delta-\dfrac{5ys}{R^2}\right) & -\alpha\dfrac{15cx}{R^7}\left(t-y+\dfrac{7yq^2}{R^2}\right) & +\alpha\dfrac{15xyz}{R^7}\\ (1-\alpha)\dfrac{3x}{R^5}(\cos2\delta-A_5\sin^2\delta) & +\alpha\dfrac{15cx}{R^7}\left(s-d+\dfrac{7dq^2}{R^2}\right) & -\alpha\dfrac{15xdz}{R^7}\end{pmatrix}$$

Here, $\cos2\delta-A_5\sin^2\delta=1-(2+A_5)\sin^2\delta$

The above three vectors correspond to the contents of the row of Tensile in Table 3.

(4) Inflation

$$u_A^o=\begin{pmatrix}-\dfrac{1-\alpha}{2}\dfrac{x}{R^3}\\ -\dfrac{1-\alpha}{2}\dfrac{y}{R^3}\\ -\dfrac{1-\alpha}{2}\dfrac{d}{R^3}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\dfrac{1-\alpha}{\alpha}\dfrac{x}{R^3}\\ \dfrac{1-\alpha}{\alpha}\dfrac{y}{R^3}\\ \dfrac{1-\alpha}{\alpha}\dfrac{d}{R^3}\end{pmatrix}\qquad u_C^o=\begin{pmatrix}(1-\alpha)\dfrac{3xd}{R^5}\\ (1-\alpha)\dfrac{3yd}{R^5}\\ (1-\alpha)\dfrac{C_3}{R^3}\end{pmatrix}\qquad C_3=1-\frac{3d^2}{R^2}$$

$$\frac{\partial u_A^o}{\partial x}=\begin{pmatrix}-\dfrac{1-\alpha}{2}\dfrac{A_3}{R^3}\\ \dfrac{1-\alpha}{2}\dfrac{3xy}{R^5}\\ \dfrac{1-\alpha}{2}\dfrac{3xd}{R^5}\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial x}=\begin{pmatrix}\dfrac{1-\alpha}{\alpha}\dfrac{A_3}{R^3}\\ -\dfrac{1-\alpha}{\alpha}\dfrac{3xy}{R^5}\\ -\dfrac{1-\alpha}{\alpha}\dfrac{3xd}{R^5}\end{pmatrix}\qquad \frac{\partial u_C^o}{\partial x}=\begin{pmatrix}(1-\alpha)\dfrac{3d}{R^5}A_5\\ -(1-\alpha)\dfrac{15xyd}{R^7}\\ -(1-\alpha)\dfrac{3x}{R^5}C_5\end{pmatrix}\qquad \begin{aligned}A_3&=1-\frac{3x^2}{R^2}\\ A_5&=1-\frac{5x^2}{R^2}\\ C_5&=1-\frac{5d^2}{R^2}\end{aligned}$$

The above three vectors correspond to the contents of the row of Inflation in Table 3.

[II] Derivation of Table 4 (y-Derivative)

Table 4 can be derived by differentiation of Table 2 with y-coordinate (refer Appendix : Table of Differentiation). In the following, the notation is matched with Tables in Okada (1992).

Displacement : $u^0(x,y,z)=\frac{M_0}{2\pi\mu}\left[u^0{}_A(x,y,z)-u^0{}_A(x,y,-z)+u^0{}_B(x,y,z)+zu^0{}_C(x,y,z)\right]$

y-Derivative : $\frac{\partial u^0}{\partial y}(x,y,z)=\frac{M_0}{2\pi\mu}\left[\frac{\partial u^0{}_A}{\partial y}(x,y,z)-\frac{\partial u^0{}_A}{\partial y}(x,y,-z)+\frac{\partial u^0{}_B}{\partial y}(x,y,z)+z\frac{\partial u^0{}_C}{\partial y}(x,y,z)\right]$

(1) Strike slip

$$u_A^0=\begin{pmatrix}\frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3x^2q}{R^5}\\ \frac{1-\alpha}{2}\frac{x}{R^3}\sin\delta+\frac{\alpha}{2}\frac{3xyq}{R^5}\\ -\frac{1-\alpha}{2}\frac{x}{R^3}\cos\delta+\frac{\alpha}{2}\frac{3xdq}{R^5}\end{pmatrix}\quad u_B^0=\begin{pmatrix}-\frac{3x^2q}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin\delta\\ -\frac{3xyq}{R^5}-\frac{1-\alpha}{\alpha}I_2^0\sin\delta\\ -\frac{3cxq}{R^5}-\frac{1-\alpha}{\alpha}I_4^0\sin\delta\end{pmatrix}\quad u_C^0=\begin{pmatrix}-(1-\alpha)\frac{A_3}{R^3}\cos\delta+\alpha\frac{3cq}{R^5}A_5\\ (1-\alpha)\frac{3xy}{R^5}\cos\delta+\alpha\frac{3cx}{R^5}\left(\sin\delta-\frac{5yq}{R^2}\right)\\ -(1-\alpha)\frac{3xy}{R^5}\sin\delta+\alpha\frac{3cx}{R^5}\left(\cos\delta+\frac{5dq}{R^2}\right)\end{pmatrix}$$

where, $d=c-z\,,\ q=y\sin\delta-d\cos\delta\,,\ R^2=x^2+y^2+d^2$ and

$$I_1^0=y\left[\frac{1}{R(R+d)^2}-x^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_2^0=x\left[\frac{1}{R(R+d)^2}-y^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_4^0=-xy\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial y}=\begin{pmatrix}\frac{1-\alpha}{2}\frac{1}{R^3}\left(\sin\delta-\frac{3yq}{R^2}\right)+\frac{\alpha}{2}\frac{3x^2}{R^5}U\\ -\frac{1-\alpha}{2}\frac{3xy}{R^5}\sin\delta+\frac{\alpha}{2}\left(\frac{3xy}{R^5}U+\frac{3xq}{R^5}\right)\\ \frac{1-\alpha}{2}\frac{3xy}{R^5}\cos\delta+\frac{\alpha}{2}\frac{3xd}{R^5}U\end{pmatrix}\quad \frac{\partial u_B^0}{\partial y}=\begin{pmatrix}-\frac{3x^2}{R^5}U-\frac{1-\alpha}{\alpha}J_2^0\sin\delta\\ -\frac{3xy}{R^5}U-\frac{3xq}{R^5}-\frac{1-\alpha}{\alpha}J_4^0\sin\delta\\ -\frac{3cx}{R^5}U-\frac{1-\alpha}{\alpha}K_2^0\sin\delta\end{pmatrix}\quad U=\sin\delta-\frac{5yq}{R^2}$$

$$\frac{\partial u_C^0}{\partial y}=\begin{pmatrix}(1-\alpha)\frac{3y}{R^5}A_5\cos\delta+\alpha\frac{3c}{R^5}\left(A_5\sin\delta-\frac{5yq}{R^2}A_7\right)\\ (1-\alpha)\frac{3x}{R^5}B_5\cos\delta-\alpha\frac{15cx}{R^7}(2y\sin\delta+qB_7)\\ -(1-\alpha)\frac{3x}{R^5}B_5\sin\delta+\alpha\frac{15cx}{R^7}\left(d\sin\delta-y\cos\delta-\frac{7ydq}{R^2}\right)\end{pmatrix}$$

$$J_2^0\equiv\frac{\partial}{\partial y}I_1^0=\frac{1}{R^3}-\frac{3}{R(R+d)^2}+3x^2y^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}$$

$$J_4^0\equiv\frac{\partial}{\partial y}I_2^0=-3xy\left[\frac{3R+d}{R^3(R+d)^3}-y^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}\right]$$

$$K_2^0\equiv\frac{\partial}{\partial y}I_4^0=-x\left[\frac{2R+d}{R^3(R+d)^2}-y^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]$$

Here, $d\sin\delta-y\cos\delta-\frac{7ydq}{R^2}=d\sin\delta-y\cos\delta-\frac{7yd}{R^2}(y\sin\delta-d\cos\delta)=d\left(1-\frac{7y^2}{R^2}\right)\sin\delta-y\left(1-\frac{7d^2}{R^2}\right)\cos\delta=dB_7\sin\delta-yC_7\cos\delta$

The above three vectors correspond to the contents of the row of Strike-Slip in Table 4.

(2) Dip slip

$$u_A^0=\begin{pmatrix}\frac{\alpha}{2}\frac{3xpq}{R^5}\\ \frac{1-\alpha}{2}\frac{s}{R^3}+\frac{\alpha}{2}\frac{3ypq}{R^5}\\ -\frac{1-\alpha}{2}\frac{t}{R^3}+\frac{\alpha}{2}\frac{3dpq}{R^5}\end{pmatrix}\quad u_B^0=\begin{pmatrix}-\frac{3xpq}{R^5}+\frac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta\\ -\frac{3ypq}{R^5}+\frac{1-\alpha}{\alpha}I_1^0\sin\delta\cos\delta\\ -\frac{3cpq}{R^5}+\frac{1-\alpha}{\alpha}I_5^0\sin\delta\cos\delta\end{pmatrix}\quad u_C^0=\begin{pmatrix}(1-\alpha)\frac{3xt}{R^5}-\alpha\frac{15cxpq}{R^7}\\ (1-\alpha)\left(\frac{3yt}{R^5}-\frac{\cos2\delta}{R^3}\right)-\alpha\frac{15cypq}{R^7}+\alpha\frac{3cs}{R^5}\\ -(1-\alpha)\frac{A_3}{R^3}\sin\delta\cos\delta+\alpha\frac{15cdpq}{R^7}+\alpha\frac{3ct}{R^5}\end{pmatrix}$$

where, $d=c-z\,,\ \begin{cases}p=y\cos\delta+d\sin\delta\\ q=y\sin\delta-d\cos\delta\end{cases},\ \begin{cases}s=p\sin\delta+q\cos\delta=y\sin2\delta-d\cos2\delta\\ t=p\cos\delta-q\sin\delta=y\cos2\delta+d\sin2\delta\end{cases}$ and

$$I_1^0=y\left[\frac{1}{R(R+d)^2}-x^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_2^0=x\left[\frac{1}{R(R+d)^2}-y^2\frac{3R+d}{R^3(R+d)^3}\right]\quad I_3^0=\frac{x}{R^3}-I_2^0\quad I_5^0=\frac{1}{R(R+d)}-x^2\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial y}=\begin{pmatrix}\frac{\alpha}{2}\frac{3x}{R^5}V\\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\sin2\delta-\frac{3ys}{R^2}\right)+\frac{\alpha}{2}\left(\frac{3y}{R^5}V+\frac{3pq}{R^5}\right)\\ -\frac{1-\alpha}{2}\frac{1}{R^3}\left(\cos2\delta-\frac{3yt}{R^2}\right)+\frac{\alpha}{2}\frac{3d}{R^5}V\end{pmatrix}\quad \frac{\partial u_B^0}{\partial y}=\begin{pmatrix}-\frac{3x}{R^5}V+\frac{1-\alpha}{\alpha}J_1^0\sin\delta\cos\delta\\ -\frac{3y}{R^5}V-\frac{3pq}{R^5}+\frac{1-\alpha}{\alpha}J_2^0\sin\delta\cos\delta\\ -\frac{3c}{R^5}V+\frac{1-\alpha}{\alpha}K_1^0\sin\delta\cos\delta\end{pmatrix}\quad V=s-\frac{5ypq}{R^2}$$

$$\frac{\partial u_C^o}{\partial y}=\begin{pmatrix}(1-\alpha)\frac{3x}{R^5}\left(\cos2\delta-\frac{5yt}{R^2}\right) & -\alpha\frac{15cx}{R^7}\left(s-\frac{7ypq}{R^2}\right)\\ (1-\alpha)\frac{3}{R^5}(2y\cos2\delta+tB_5)+\alpha\frac{3c}{R^5}\left(\sin2\delta-\frac{10ys}{R^2}-\frac{5pq}{R^2}B_7\right) & \\ (1-\alpha)\frac{3y}{R^5}A_5\sin\delta\cos\delta-\alpha\frac{3c}{R^5}\left(-\cos2\delta+\frac{5(yt-ds)}{R^2}+\frac{35ydpq}{R^4}\right) & \end{pmatrix}$$

$$J_1^0\equiv\frac{\partial}{\partial y}I_3^0=-\frac{3xy}{R^5}-J_4^0=-3xy\left[\frac{3R+d}{R^3(R+d)^3}-x^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}\right]$$

$$J_2^0\equiv\frac{\partial}{\partial y}I_1^0=\frac{1}{R^3}-\frac{3}{R(R+d)^2}+3x^2y^2\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}$$

$$K_1^0\equiv\frac{\partial}{\partial y}I_5^0=-y\left[\frac{2R+d}{R^3(R+d)^2}-x^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]$$

Here, $\frac{5(yt-ds)}{R^2}=\frac{5(y^2+d^2)}{R^2}\cos2\delta=\frac{5(R^2-x^2)}{R^2}\cos2\delta=(4+A_5)\cos2\delta$

The above three vectors correspond to the contents of the row of Dip-Slip in Table 4.

(3) Tensile

$$u_A^o=\begin{pmatrix}\frac{1-\alpha}{2}\frac{x}{R^3}-\frac{\alpha}{2}\frac{3xq^2}{R^5}\\ \frac{1-\alpha}{2}\frac{t}{R^3}-\frac{\alpha}{2}\frac{3yq^2}{R^5}\\ \frac{1-\alpha}{2}\frac{s}{R^3}-\frac{\alpha}{2}\frac{3dq^2}{R^5}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\frac{3xq^2}{R^5}-\frac{1-\alpha}{\alpha}I_3^0\sin^2\delta\\ \frac{3yq^2}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin^2\delta\\ \frac{3cq^2}{R^5}-\frac{1-\alpha}{\alpha}I_5^0\sin^2\delta\end{pmatrix}$$

$$u_C^o=\begin{pmatrix}-(1-\alpha)\frac{3xs}{R^5}+\alpha\frac{15cxq^2}{R^7}-\alpha\frac{3xz}{R^5}\\ (1-\alpha)\left(\frac{\sin2\delta}{R^3}-\frac{3ys}{R^5}\right)+3c\alpha\frac{t-y}{R^5}+\alpha\frac{15cyq^2}{R^7}-\alpha\frac{3yz}{R^5}\\ -(1-\alpha)\left(\frac{\cos^2\delta}{R^3}+\frac{3x^2}{R^5}\sin^2\delta\right)-3c\alpha\frac{s-d}{R^5}-\alpha\frac{15cdq^2}{R^7}+\alpha\frac{3dz}{R^5}\end{pmatrix}$$

$$\frac{\partial u_A^o}{\partial y}=\begin{pmatrix}-\frac{1-\alpha}{2}\frac{3xy}{R^5}-\frac{\alpha}{2}\frac{3xq}{R^5}W\\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\cos2\delta-\frac{3yt}{R^2}\right)-\frac{\alpha}{2}\left(\frac{3yq}{R^5}W+\frac{3q^2}{R^5}\right)\\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\sin2\delta-\frac{3ys}{R^2}\right)-\frac{\alpha}{2}\frac{3dq}{R^5}W\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial y}=\begin{pmatrix}\frac{3xq}{R^5}W-\frac{1-\alpha}{\alpha}J_1^0\sin^2\delta\\ \frac{3yq}{R^5}W+\frac{3q^2}{R^5}-\frac{1-\alpha}{\alpha}J_2^0\sin^2\delta\\ \frac{3cq}{R^5}W-\frac{1-\alpha}{\alpha}K_1^0\sin^2\delta\end{pmatrix}\qquad W=2\sin\delta-\frac{5yq}{R^2}$$

$$\frac{\partial u_C^o}{\partial y}=\begin{pmatrix}-(1-\alpha)\frac{3x}{R^5}\left(\sin2\delta-\frac{5ys}{R^2}\right)-\alpha\frac{15cx}{R^7}\left(-2q\sin\delta+\frac{7yq^2}{R^2}\right)+\alpha\frac{15xyz}{R^7}\\ -(1-\alpha)\frac{3}{R^5}(2y\sin2\delta+sB_5)-\alpha\frac{3c}{R^5}\left[2\sin^2\delta+\frac{5y}{R^2}(t-y-2q\sin\delta)-\frac{5q^2}{R^2}B_7\right]-\alpha\frac{3z}{R^5}B_5\\ (1-\alpha)\frac{3y}{R^5}(1-A_5\sin^2\delta)+\alpha\frac{3c}{R^5}\left[-\sin2\delta+\frac{5(ys+dt)}{R^2}-\frac{5yd}{R^2}\left(2-\frac{7q^2}{R^2}\right)\right]-\alpha\frac{15ydz}{R^7}\end{pmatrix}$$

Here, since $\begin{cases}y=p\cos\delta+q\sin\delta\\ t=p\cos\delta-q\sin\delta\end{cases}$, $\quad -2q\sin\delta=t-y$

and $\frac{5(ys+dt)}{R^2}=\frac{5(y^2+d^2)}{R^2}\sin2\delta=\frac{5(R^2-x^2)}{R^2}\sin2\delta=(4+A_5)\sin2\delta$

The above three vectors correspond to the contents of the row of Tensile in Table 4.

(4) Inflation

$$u_A^o=\begin{pmatrix}-\frac{1-\alpha}{2}\frac{x}{R^3}\\ -\frac{1-\alpha}{2}\frac{y}{R^3}\\ -\frac{1-\alpha}{2}\frac{d}{R^3}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\frac{1-\alpha}{\alpha}\frac{x}{R^3}\\ \frac{1-\alpha}{\alpha}\frac{y}{R^3}\\ \frac{1-\alpha}{\alpha}\frac{d}{R^3}\end{pmatrix}\qquad u_C^o=\begin{pmatrix}(1-\alpha)\frac{3xd}{R^5}\\ (1-\alpha)\frac{3yd}{R^5}\\ (1-\alpha)\frac{C_3}{R^3}\end{pmatrix}\qquad C_3=1-\frac{3d^2}{R^2}$$

$$\frac{\partial u_A^o}{\partial y}=\begin{pmatrix}\frac{1-\alpha}{2}\frac{3xy}{R^5}\\ -\frac{1-\alpha}{2}\frac{B_3}{R^3}\\ \frac{1-\alpha}{2}\frac{3yd}{R^5}\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial y}=\begin{pmatrix}-\frac{1-\alpha}{\alpha}\frac{3xy}{R^5}\\ \frac{1-\alpha}{\alpha}\frac{B_3}{R^3}\\ -\frac{1-\alpha}{\alpha}\frac{3yd}{R^5}\end{pmatrix}\qquad \frac{\partial u_C^o}{\partial y}=\begin{pmatrix}-(1-\alpha)\frac{15xyd}{R^7}\\ (1-\alpha)\frac{3d}{R^5}B_5\\ -(1-\alpha)\frac{3y}{R^5}C_5\end{pmatrix}$$

$$B_3=1-\frac{3y^2}{R^2}\qquad B_5=1-\frac{5y^2}{R^2}\qquad C_5=1-\frac{5d^2}{R^2}$$

The above three vectors correspond to the contents of the row of Inflation in Table 4.

[III] Derivation of Table 5 (z-Derivative)

Table 5 can be derived by differentiation of Table 2 with z-coordinate (refer Appendix : Table of Differentiation). In the following, the notation is matched with Tables in Okada (1992).

Displacement : $u^0(x,y,z) = \frac{M_0}{2\pi\mu}[u^0{}_A(x,y,z) - u^0{}_A(x,y,-z) + u^0{}_B(x,y,z) + zu^0{}_C(x,y,z)]$

z-Derivative : $\frac{\partial u^0}{\partial z}(x,y,z) = \frac{M_0}{2\pi\mu}\left[\frac{\partial u^0{}_A}{\partial z}(x,y,z) + \frac{\partial u^0{}_A}{\partial z}(x,y,-z) + \frac{\partial u^0{}_B}{\partial z}(x,y,z) + u^0(x,y,z) + z\frac{\partial u^0{}_C}{\partial z}(x,y,z)\right]$

(1) Strike slip

$$u_A^0 = \begin{pmatrix} \frac{1-\alpha}{2}\frac{q}{R^3} + \frac{\alpha}{2}\frac{3x^2q}{R^5} \\ \frac{1-\alpha}{2}\frac{x}{R^3}\sin\delta + \frac{\alpha}{2}\frac{3xyq}{R^5} \\ -\frac{1-\alpha}{2}\frac{x}{R^3}\cos\delta + \frac{\alpha}{2}\frac{3xdq}{R^5} \end{pmatrix} \quad u_B^0 = \begin{pmatrix} -\frac{3x^2q}{R^5} - \frac{1-\alpha}{\alpha}I_1^0\sin\delta \\ -\frac{3xyq}{R^5} - \frac{1-\alpha}{\alpha}I_2^0\sin\delta \\ -\frac{3cxq}{R^5} - \frac{1-\alpha}{\alpha}I_4^0\sin\delta \end{pmatrix} \quad u_C^0 = \begin{pmatrix} -(1-\alpha)\frac{A_3}{R^3}\cos\delta + \alpha\frac{3cq}{R^5}A_5 \\ (1-\alpha)\frac{3xy}{R^5}\cos\delta + \alpha\frac{3cx}{R^5}\left(\sin\delta - \frac{5yq}{R^2}\right) \\ -(1-\alpha)\frac{3xy}{R^5}\sin\delta + \alpha\frac{3cx}{R^5}\left(\cos\delta + \frac{5dq}{R^2}\right) \end{pmatrix}$$

where, $d = c - z,\ q = y\sin\delta - d\cos\delta,\ R^2 = x^2 + y^2 + d^2$ and

$$I_1^0 = y\left[\frac{1}{R(R+d)^2} - x^2\frac{3R+d}{R^3(R+d)^3}\right] \quad I_2^0 = x\left[\frac{1}{R(R+d)^2} - y^2\frac{3R+d}{R^3(R+d)^3}\right] \quad I_4^0 = -xy\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial z} = \begin{pmatrix} \frac{1-\alpha}{2}\frac{1}{R^3}\left(\cos\delta + \frac{3dq}{R^2}\right) + \frac{\alpha}{2}\frac{3x^2}{R^5}U' \\ \frac{1-\alpha}{2}\frac{3xd}{R^5}\sin\delta + \frac{\alpha}{2}\frac{3xy}{R^5}U' \\ -\frac{1-\alpha}{2}\frac{3xd}{R^5}\cos\delta + \frac{\alpha}{2}\left(\frac{3xd}{R^5}U' - \frac{3xq}{R^5}\right) \end{pmatrix} \quad \frac{\partial u_B^0}{\partial z} = \begin{pmatrix} -\frac{3x^2}{R^5}U' + \frac{1-\alpha}{\alpha}K_1^0\sin\delta \\ -\frac{3xy}{R^5}U' + \frac{1-\alpha}{\alpha}K_2^0\sin\delta \\ -\frac{3cx}{R^5}U' + \frac{1-\alpha}{\alpha}K_4^0\sin\delta \end{pmatrix} \quad U' = \cos\delta + \frac{5dq}{R^2}$$

$$\frac{\partial u_C^0}{\partial z} = \begin{pmatrix} -(1-\alpha)\frac{3d}{R^5}A_5\cos\delta + \alpha\frac{3c}{R^5}\left(A_5\cos\delta + \frac{5dq}{R^2}A_7\right) \\ (1-\alpha)\frac{15xyd}{R^7}\cos\delta + \alpha\frac{15cx}{R^7}\left(d\sin\delta - y\cos\delta - \frac{7ydq}{R^2}\right) \\ -(1-\alpha)\frac{15xyd}{R^7}\sin\delta + \alpha\frac{15cx}{R^7}(2d\cos\delta - qC_7) \end{pmatrix}$$

$$K_1^0 \equiv -\frac{\partial}{\partial z}I_1^0 = -y\left[\frac{2R+d}{R^3(R+d)^2} - x^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]$$

$$K_2^0 \equiv -\frac{\partial}{\partial z}I_2^0 = -x\left[\frac{2R+d}{R^3(R+d)^2} - y^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]$$

$$K_4^0 \equiv -\frac{\partial}{\partial z}I_4^0 = \frac{3xy}{R^5}$$

Here, $d\sin\delta - y\cos\delta - \frac{7ydq}{R^2} = d\sin\delta - y\cos\delta - \frac{7yd}{R^2}(y\sin\delta - d\cos\delta) = d\left(1 - \frac{7y^2}{R^2}\right)\sin\delta - y\left(1 - \frac{7d^2}{R^2}\right)\cos\delta = dB_7\sin\delta - yC_7\cos\delta$

The above three vectors correspond to the contents of the row of Strike-Slip in Table 5.

(2) Dip slip

$$u_A^0 = \begin{pmatrix} \frac{\alpha}{2}\frac{3xpq}{R^5} \\ \frac{1-\alpha}{2}\frac{s}{R^3} + \frac{\alpha}{2}\frac{3ypq}{R^5} \\ -\frac{1-\alpha}{2}\frac{t}{R^3} + \frac{\alpha}{2}\frac{3dpq}{R^5} \end{pmatrix} \quad u_B^0 = \begin{pmatrix} -\frac{3xpq}{R^5} + \frac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta \\ -\frac{3ypq}{R^5} + \frac{1-\alpha}{\alpha}I_1^0\sin\delta\cos\delta \\ -\frac{3cpq}{R^5} + \frac{1-\alpha}{\alpha}I_5^0\sin\delta\cos\delta \end{pmatrix} \quad u_C^0 = \begin{pmatrix} (1-\alpha)\frac{3xt}{R^5} - \alpha\frac{15cxpq}{R^7} \\ (1-\alpha)\left(\frac{3yt}{R^5} - \frac{\cos 2\delta}{R^3}\right) - \alpha\frac{15cypq}{R^7} + \alpha\frac{3cs}{R^5} \\ -(1-\alpha)\frac{A_3}{R^3}\sin\delta\cos\delta + \alpha\frac{15cdpq}{R^7} + \alpha\frac{3ct}{R^5} \end{pmatrix}$$

where, $d = c - z,\ \begin{cases} p = y\cos\delta + d\sin\delta \\ q = y\sin\delta - d\cos\delta \end{cases},\ \begin{cases} s = p\sin\delta + q\cos\delta = y\sin 2\delta - d\cos 2\delta \\ t = p\cos\delta - q\sin\delta = y\cos 2\delta + d\sin 2\delta \end{cases}$ and

$$I_1^0 = y\left[\frac{1}{R(R+d)^2} - x^2\frac{3R+d}{R^3(R+d)^3}\right] \quad I_2^0 = x\left[\frac{1}{R(R+d)^2} - y^2\frac{3R+d}{R^3(R+d)^3}\right] \quad I_3^0 = \frac{x}{R^3} - I_2^0 \quad I_5^0 = \frac{1}{R(R+d)} - x^2\frac{2R+d}{R^3(R+d)^2}$$

$$\frac{\partial u_A^0}{\partial z} = \begin{pmatrix} \frac{\alpha}{2}\frac{3x}{R^5}V' \\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\cos 2\delta + \frac{3ds}{R^2}\right) + \frac{\alpha}{2}\frac{3y}{R^5}V' \\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\sin 2\delta - \frac{3dt}{R^2}\right) + \frac{\alpha}{2}\left(\frac{3d}{R^5}V' - \frac{3pq}{R^5}\right) \end{pmatrix} \quad V' = t + \frac{5dpq}{R^2}$$

$$\frac{\partial u_B^0}{\partial z} = \begin{pmatrix} -\frac{3x}{R^5}V' - \frac{1-\alpha}{\alpha}K_3^0\sin\delta\cos\delta \\ -\frac{3y}{R^5}V' - \frac{1-\alpha}{\alpha}K_1^0\sin\delta\cos\delta \\ -\frac{3c}{R^5}V' - \frac{1-\alpha}{\alpha}K_5^0\sin\delta\cos\delta \end{pmatrix}$$

$$K_3^0 \equiv -\frac{\partial}{\partial z}I_3^0 = -\frac{3xd}{R^5} - K_2^0$$

$$K_1^0 \equiv -\frac{\partial}{\partial z}I_1^0 = -y\left[\frac{2R+d}{R^3(R+d)^2} - x^2\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}\right]$$

$$K_5^0 \equiv -\frac{\partial}{\partial z}I_5^0 = -\frac{A_3}{R^3}$$

$$\frac{\partial u_C^o}{\partial z}=\begin{pmatrix}-(1-\alpha)\frac{3x}{R^5}\left(\sin 2\delta-\frac{5dt}{R^2}\right) & -\alpha\frac{15cx}{R^7}\left(t+\frac{7dpq}{R^2}\right)\\ -(1-\alpha)\frac{3}{R^5}\left[y\sin 2\delta+d\cos 2\delta-\frac{5ydt}{R^2}\right] & -\alpha\frac{3c}{R^5}\left(-\cos 2\delta+\frac{5(yt-ds)}{R^2}+\frac{35ydpq}{R^4}\right)\\ -(1-\alpha)\frac{3d}{R^5}A_5\sin\delta\cos\delta & -\alpha\frac{3c}{R^5}\left(\sin 2\delta-\frac{10dt}{R^2}+\frac{5pq}{R^2}C_7\right)\end{pmatrix}$$

Here, $y\sin 2\delta+d\cos 2\delta-\frac{5ydt}{R^2}=y\sin 2\delta+d\cos 2\delta-\frac{5yd}{R^2}(y\cos 2\delta+d\sin 2\delta)=dB_5\cos 2\delta+yC_5\sin 2\delta$

and $\frac{5(yt-ds)}{R^2}=\frac{5(y^2+d^2)}{R^2}\cos 2\delta=\frac{5(R^2-x^2)}{R^2}\cos 2\delta=(4+A_5)\cos 2\delta$

The above three vectors correspond to the contents of the row of Dip-Slip in Table 5.

(3) Tensile

$$u_A^o=\begin{pmatrix}\frac{1-\alpha}{2}\frac{x}{R^3}-\frac{\alpha}{2}\frac{3xq^2}{R^5}\\ \frac{1-\alpha}{2}\frac{t}{R^3}-\frac{\alpha}{2}\frac{3yq^2}{R^5}\\ \frac{1-\alpha}{2}\frac{s}{R^3}-\frac{\alpha}{2}\frac{3dq^2}{R^5}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\frac{3xq^2}{R^5}-\frac{1-\alpha}{\alpha}I_3^0\sin^2\delta\\ \frac{3yq^2}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin^2\delta\\ \frac{3cq^2}{R^5}-\frac{1-\alpha}{\alpha}I_5^0\sin^2\delta\end{pmatrix}$$

$$u_C^o=\begin{pmatrix}-(1-\alpha)\frac{3xs}{R^5} & +\alpha\frac{15cxq^2}{R^7}-\alpha\frac{3xz}{R^5}\\ (1-\alpha)\left(\frac{\sin 2\delta}{R^3}-\frac{3ys}{R^5}\right) & +3c\alpha\frac{t-y}{R^5}+\alpha\frac{15cyq^2}{R^7}-\alpha\frac{3yz}{R^5}\\ -(1-\alpha)\left(\frac{\cos^2\delta}{R^3}+\frac{3x^2}{R^5}\sin^2\delta\right) & -3c\alpha\frac{s-d}{R^5}-\alpha\frac{15cdq^2}{R^7}+\alpha\frac{3dz}{R^5}\end{pmatrix}$$

$$\frac{\partial u_A^o}{\partial z}=\begin{pmatrix}\frac{1-\alpha}{2}\frac{3xd}{R^5} & -\frac{\alpha}{2}\frac{3xq}{R^5}W'\\ -\frac{1-\alpha}{2}\frac{1}{R^3}\left(\sin 2\delta-\frac{3dt}{R^2}\right) & -\frac{\alpha}{2}\frac{3yq}{R^5}W'\\ \frac{1-\alpha}{2}\frac{1}{R^3}\left(\cos 2\delta+\frac{3ds}{R^2}\right) & -\frac{\alpha}{2}\left(\frac{3dq}{R^5}W'-\frac{3q^2}{R^5}\right)\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial z}=\begin{pmatrix}\frac{3xq}{R^5}W'+\frac{1-\alpha}{\alpha}K_3^0\sin^2\delta\\ \frac{3yq}{R^5}W'+\frac{1-\alpha}{\alpha}K_1^0\sin^2\delta\\ \frac{3cq}{R^5}W'+\frac{1-\alpha}{\alpha}K_5^0\sin^2\delta\end{pmatrix}\qquad W'=2\cos\delta+\frac{5dq}{R^2}$$

$$\frac{\partial u_C^o}{\partial z}=\begin{pmatrix}-(1-\alpha)\frac{3x}{R^5}\left(\cos 2\delta+\frac{5ds}{R^2}\right) & +\alpha\frac{15cx}{R^7}\left(2q\cos\delta+\frac{7dq^2}{R^2}\right)-\alpha\frac{3x}{R^5}\left(1+\frac{5dz}{R^2}\right)\\ (1-\alpha)\frac{3}{R^5}\left(d\sin 2\delta-y\cos 2\delta-\frac{5yds}{R^2}\right) & +\alpha\frac{3c}{R^5}\left[-\sin 2\delta+\frac{5d(t-y)}{R^2}+\frac{5y}{R^2}\left(2q\cos\delta+\frac{7dq^2}{R^2}\right)\right]-\alpha\frac{3y}{R^5}\left(1+\frac{5dz}{R^2}\right)\\ -(1-\alpha)\frac{3d}{R^5}(1-A_5\sin^2\delta) & -\alpha\frac{3c}{R^5}\left[1+\cos 2\delta+\frac{5d(s-d)}{R^2}+\frac{5}{R^2}(2dq\cos\delta-q^2C_7)\right]+\alpha\frac{3c}{R^5}-\alpha\frac{3z}{R^5}(1+C_5)\end{pmatrix}$$

Here, $d\sin 2\delta-y\cos 2\delta-\frac{5yds}{R^2}=d\sin 2\delta-y\cos 2\delta-\frac{5yd}{R^2}(y\sin 2\delta-d\cos 2\delta)=dB_5\sin 2\delta-yC_5\cos 2\delta$

Since $\begin{cases}d=p\sin\delta-q\cos\delta\\ s=p\sin\delta+q\cos\delta\end{cases}$, $\quad 2q\cos\delta=s-d$

So, $\frac{5d(t-y)}{R^2}+\frac{5y}{R^2}\left(2q\cos\delta+\frac{7dq^2}{R^2}\right)=\frac{5(ys+dt)}{R^2}-\frac{5yd}{R^2}\left(2-\frac{7q^2}{R^2}\right)=\frac{5(R^2-x^2)}{R^2}\sin 2\delta-\frac{5yd}{R^2}\left(2-\frac{7q^2}{R^2}\right)=(4+A_5)\sin 2\delta-\frac{5yd}{R^2}\left(2-\frac{7q^2}{R^2}\right)$

The above three vectors correspond to the contents of the row of Tensile in Table 5.

(4) Inflation

$$u_A^o=\begin{pmatrix}-\frac{1-\alpha}{2}\frac{x}{R^3}\\ -\frac{1-\alpha}{2}\frac{y}{R^3}\\ -\frac{1-\alpha}{2}\frac{d}{R^3}\end{pmatrix}\qquad u_B^o=\begin{pmatrix}\frac{1-\alpha}{\alpha}\frac{x}{R^3}\\ \frac{1-\alpha}{\alpha}\frac{y}{R^3}\\ \frac{1-\alpha}{\alpha}\frac{d}{R^3}\end{pmatrix}\qquad u_C^o=\begin{pmatrix}(1-\alpha)\frac{3xd}{R^5}\\ (1-\alpha)\frac{3yd}{R^5}\\ (1-\alpha)\frac{C_3}{R^3}\end{pmatrix}$$

$$\frac{\partial u_A^o}{\partial z}=\begin{pmatrix}-\frac{1-\alpha}{2}\frac{3xd}{R^5}\\ -\frac{1-\alpha}{2}\frac{3yd}{R^5}\\ \frac{1-\alpha}{2}\frac{C_3}{R^3}\end{pmatrix}\qquad \frac{\partial u_B^o}{\partial z}=\begin{pmatrix}\frac{1-\alpha}{\alpha}\frac{3xd}{R^5}\\ \frac{1-\alpha}{\alpha}\frac{3yd}{R^5}\\ -\frac{1-\alpha}{\alpha}\frac{C_3}{R^3}\end{pmatrix}\qquad \frac{\partial u_C^o}{\partial z}=\begin{pmatrix}-(1-\alpha)\frac{3x}{R^5}C_5\\ -(1-\alpha)\frac{3y}{R^5}C_5\\ (1-\alpha)\frac{3d}{R^5}(2+C_5)\end{pmatrix}\qquad \begin{aligned}C_3&=1-\frac{3d^2}{R^2}\\ C_5&=1-\frac{5d^2}{R^2}\end{aligned}$$

The above three vectors correspond to the contents of the row of Inflation in Table 5.

Appendix : Table of Differentiation

$R=\sqrt{x^2+y^2+d^2}$, $d=c-z$, $\begin{cases} p = y\cos\delta + d\sin\delta \\ q = y\sin\delta - d\cos\delta \end{cases}$, $\begin{cases} s = p\sin\delta + q\cos\delta = y\sin2\delta - d\cos2\delta \\ t = p\cos\delta - q\sin\delta = y\cos2\delta + d\sin2\delta \end{cases}$

f	$\partial f/\partial x$	$\partial f/\partial y$	$\partial f/\partial z = -\partial f/\partial d$
$1/R^3$	$-3x/R^5$	$-3y/R^5$	$3d/R^5$
x/R^3	A_3/R^3	$-3xy/R^5$	$3xd/R^5$
y/R^3	$-3xy/R^5$	B_3/R^3	$3yd/R^5$
d/R^3	$-3xd/R^5$	$-3yd/R^5$	$-C_3/R^3$
q/R^3	$-3xq/R^5$	$\frac{1}{R^3}\left(\sin\delta - \frac{3yq}{R^2}\right)$	$\frac{1}{R^3}\left(\cos\delta + \frac{3dq}{R^2}\right)$
s/R^3	$-3xs/R^5$	$\frac{1}{R^3}\left(\sin2\delta - \frac{3ys}{R^2}\right)$	$\frac{1}{R^3}\left(\cos2\delta + \frac{3ds}{R^2}\right)$
t/R^3	$-3xt/R^5$	$\frac{1}{R^3}\left(\cos2\delta - \frac{3yt}{R^2}\right)$	$-\frac{1}{R^3}\left(\sin2\delta - \frac{3dt}{R^2}\right)$
$\frac{A_3}{R^3} = \frac{1}{R^3} - \frac{3x^2}{R^5}$	$-\frac{3x}{R^5}(2+A_5)$	$-\frac{3y}{R^5}A_5$	$\frac{3d}{R^5}A_5$
$1/R^5$	$-5x/R^7$	$-5y/R^7$	$5d/R^7$
x/R^5	A_5/R^5	$-5xy/R^7$	$5xd/R^7$
y/R^5	$-5xy/R^7$	B_5/R^5	$5yd/R^7$
d/R^5	$-5xd/R^7$	$-5yd/R^7$	$-C_5/R^5$
q/R^5	$-5xq/R^7$	$\frac{1}{R^5}\left(\sin\delta - \frac{5yq}{R^2}\right) = \frac{U}{R^5}$	$\frac{1}{R^5}\left(\cos\delta + \frac{5dq}{R^2}\right) = \frac{U'}{R^5}$
s/R^5	$-5xs/R^7$	$\frac{1}{R^5}\left(\sin2\delta - \frac{5ys}{R^2}\right)$	$\frac{1}{R^5}\left(\cos2\delta + \frac{5ds}{R^2}\right)$
t/R^5	$-5xt/R^7$	$\frac{1}{R^5}\left(\cos2\delta - \frac{5yt}{R^2}\right)$	$-\frac{1}{R^5}\left(\sin2\delta - \frac{5dt}{R^2}\right)$
x^2/R^5	$\frac{x}{R^5}(1+A_5)$	$-5x^2y/R^7$	$5x^2d/R^7$
y^2/R^5	$-5xy^2/R^7$	$\frac{y}{R^5}(1+B_5)$	$5y^2d/R^7$
d^2/R^5	$-5xd^2/R^7$	$-5yd^2/R^7$	$-\frac{d}{R^5}(1+C_5)$
xy/R^5	$\frac{y}{R^5}A_5$	$\frac{x}{R^5}B_5$	$5xyd/R^7$
xd/R^5	$\frac{d}{R^5}A_5$	$-5xyd/R^7$	$-\frac{x}{R^5}C_5$
yd/R^5	$-5xyd/R^7$	$\frac{d}{R^5}B_5$	$-\frac{x}{R^5}C_5$
xq/R^5	$\frac{q}{R^5}A_5$	$\frac{x}{R^5}\left(\sin\delta - \frac{5yq}{R^2}\right) = \frac{x}{R^5}U$	$\frac{x}{R^5}\left(\cos\delta + \frac{5dq}{R^2}\right) = \frac{x}{R^5}U'$
pq/R^5	$-5xpq/R^7$	$\frac{1}{R^5}\left(s - \frac{5ypq}{R^2}\right) = \frac{V}{R^5}$	$\frac{1}{R^5}\left(t + \frac{5dpq}{R^2}\right) = \frac{V'}{R^5}$
q^2/R^5	$-5xq^2/R^7$	$\frac{q}{R^5}\left(2\sin\delta - \frac{5yq}{R^2}\right) = \frac{q}{R^5}W$	$\frac{q}{R^5}\left(2\cos\delta + \frac{5dq}{R^2}\right) = \frac{q}{R^5}W'$
xs/R^5	$\frac{s}{R^5}A_5$	$\frac{x}{R^5}\left(\sin2\delta - \frac{5ys}{R^2}\right)$	$\frac{x}{R^5}\left(\cos2\delta + \frac{5ds}{R^2}\right)$
xt/R^5	$\frac{t}{R^5}A_5$	$\frac{x}{R^5}\left(\cos2\delta - \frac{5yt}{R^2}\right)$	$-\frac{x}{R^5}\left(\sin2\delta - \frac{5dt}{R^2}\right)$
ys/R^5	$-5xys/R^7$	$\frac{1}{R^5}(y\sin2\delta + sB_5)$	$\frac{y}{R^5}\left(\cos2\delta + \frac{5ds}{R^2}\right)$
yt/R^5	$-5xyt/R^7$	$\frac{1}{R^5}(y\cos2\delta + tB_5)$	$-\frac{y}{R^5}\left(\sin2\delta - \frac{5dt}{R^2}\right)$
xz/R^5	$\frac{z}{R^5}A_5$	$-5xyz/R^7$	$\frac{x}{R^5}\left(1 + \frac{5dz}{R^2}\right)$
yz/R^5	$-5xyz/R^7$	$\frac{z}{R^5}B_5$	$\frac{y}{R^5}\left(1 + \frac{5dz}{R^2}\right)$
dz/R^5	$-5xdz/R^7$	$-5ydz/R^7$	$\frac{1}{R^5}(d - zC_5)$

f	$\partial f/\partial x$	$\partial f/\partial y$	$\partial f/\partial z = -\partial f/\partial d$
x^2q/R^5	$\frac{xq}{R^5}(1+A_5)$	$\frac{x^2}{R^5}\left(\sin\delta - \frac{5yq}{R^2}\right) = \frac{x^2}{R^5}U$	$\frac{x^2}{R^5}\left(\cos\delta + \frac{5dq}{R^2}\right) = \frac{x^2}{R^5}U'$
xyq/R^5	$\frac{yq}{R^5}A_5$	$\frac{x}{R^5}(y\sin\delta + qB_5) = \frac{xy}{R^5}U + \frac{xq}{R^5}$	$\frac{xy}{R^5}\left(\cos\delta + \frac{5dq}{R^2}\right) = \frac{xy}{R^5}U'$
xdq/R^5	$\frac{dq}{R^5}A_5$	$\frac{xd}{R^5}\left(\sin\delta - \frac{5yq}{R^2}\right) = \frac{xd}{R^5}U$	$\frac{x}{R^5}(d\cos\delta - qC_5) = \frac{xd}{R^5}U' - \frac{xq}{R^5}$
xpq/R^5	$\frac{pq}{R^5}A_5$	$\frac{x}{R^5}\left(s - \frac{5ypq}{R^2}\right) = \frac{x}{R^5}V$	$\frac{x}{R^5}\left(t + \frac{5dpq}{R^2}\right) = \frac{x}{R^5}V'$
ypq/R^5	$-5xypq/R^7$	$\frac{1}{R^5}(ys + pqB_5) = \frac{y}{R^5}V + \frac{pq}{R^5}$	$\frac{y}{R^5}\left(t + \frac{5dpq}{R^2}\right) = \frac{y}{R^5}V'$
dpq/R^5	$-5xdpq/R^7$	$\frac{d}{R^5}\left(s - \frac{5ypq}{R^2}\right) = \frac{d}{R^5}V$	$\frac{1}{R^5}(dt - pqC_5) = \frac{d}{R^5}V' - \frac{pq}{R^5}$
xq^2/R^5	$\frac{q^2}{R^5}A_5$	$\frac{xq}{R^5}\left(2\sin\delta - \frac{5yq}{R^2}\right) = \frac{xq}{R^5}W$	$\frac{xq}{R^5}\left(2\cos\delta + \frac{5dq}{R^2}\right) = \frac{xq}{R^5}W'$
yq^2/R^5	$-5xyq^2/R^7$	$\frac{q}{R^5}(2y\sin\delta + qB_5) = \frac{yq}{R^5}W + \frac{q^2}{R^5}$	$\frac{yq}{R^5}\left(2\cos\delta + \frac{5dq}{R^2}\right) = \frac{yq}{R^5}W'$
dq^2/R^5	$-5xdq^2/R^7$	$\frac{dq}{R^5}\left(2\sin\delta - \frac{5yq}{R^2}\right) = \frac{xq}{R^5}W$	$\frac{q}{R^5}(2d\cos\delta - qC_5) = \frac{dq}{R^5}W' - \frac{q^2}{R^5}$
x^2q/R^7	$\frac{xq}{R^7}(1+A_7)$	$\frac{x^2}{R^7}\left(\sin\delta - \frac{7yq}{R^2}\right)$	$\frac{x^2}{R^7}\left(\cos\delta + \frac{7dq}{R^2}\right)$
xyq/R^7	$\frac{yq}{R^7}A_7$	$\frac{x}{R^7}(y\sin\delta + qB_7)$	$\frac{xy}{R^7}\left(\cos\delta + \frac{7dq}{R^2}\right)$
xdq/R^7	$\frac{dq}{R^7}A_7$	$\frac{xd}{R^7}\left(\sin\delta - \frac{7yq}{R^2}\right)$	$\frac{x}{R^7}(d\cos\delta - qC_7)$
xpq/R^7	$\frac{pq}{R^7}A_7$	$\frac{x}{R^7}\left(s - \frac{7ypq}{R^2}\right)$	$\frac{x}{R^7}\left(t + \frac{7dpq}{R^2}\right)$
ypq/R^7	$-7xypq/R^9$	$\frac{1}{R^7}(ys + pqB_7)$	$\frac{y}{R^7}\left(t + \frac{7dpq}{R^2}\right)$
dpq/R^7	$-7xdpq/R^9$	$\frac{d}{R^7}\left(s - \frac{7ypq}{R^2}\right)$	$\frac{1}{R^7}(dt - pqC_7)$
xq^2/R^7	$\frac{q^2}{R^7}A_7$	$\frac{xq}{R^7}\left(2\sin\delta - \frac{7yq}{R^2}\right)$	$\frac{xq}{R^7}\left(2\cos\delta + \frac{7dq}{R^2}\right)$
yq^2/R^7	$-7xyq^2/R^9$	$\frac{q}{R^7}(2y\sin\delta + qB_7)$	$\frac{yq}{R^7}\left(2\cos\delta + \frac{7dq}{R^2}\right)$
dq^2/R^7	$-7xdq^2/R^9$	$\frac{dq}{R^7}\left(2\sin\delta - \frac{7yq}{R^2}\right)$	$\frac{q}{R^7}(2d\cos\delta - qC_7)$
$\frac{1}{R(R+d)}$	$-x\frac{2R+d}{R^3(R+d)^2}$	$-y\frac{2R+d}{R^3(R+d)^2}$	$\frac{1}{R^3}$
$\frac{1}{R(R+d)^2}$	$-x\frac{3R+d}{R^3(R+d)^3}$	$-y\frac{3R+d}{R^3(R+d)^3}$	$\frac{2R+d}{R^3(R+d)^2}$
$\frac{2R+d}{R^3(R+d)^2}$	$-x\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}$	$-y\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}$	$\frac{3}{R^5}$
$\frac{3R+d}{R^3(R+d)^3}$	$-3x\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}$	$-3y\frac{5R^2+4Rd+d^2}{R^5(R+d)^4}$	$\frac{8R^2+9Rd+3d^2}{R^5(R+d)^3}$

$A_3 = 1 - \frac{3x^2}{R^2}$ $\quad B_3 = 1 - \frac{3y^2}{R^2}$ $\quad C_3 = 1 - \frac{3d^2}{R^2}$ $\quad U = \sin\delta - \frac{5yq}{R^2}$ $\quad U' = \cos\delta + \frac{5dq}{R^2}$

$A_5 = 1 - \frac{5x^2}{R^2}$ $\quad B_5 = 1 - \frac{5y^2}{R^2}$ $\quad C_5 = 1 - \frac{5d^2}{R^2}$ $\quad V = s - \frac{5ypq}{R^2}$ $\quad V' = t + \frac{5dpq}{R^2}$

$A_7 = 1 - \frac{7x^2}{R^2}$ $\quad B_7 = 1 - \frac{7y^2}{R^2}$ $\quad C_7 = 1 - \frac{7d^2}{R^2}$ $\quad W = 2\sin\delta - \frac{5yq}{R^2}$ $\quad W' = 2\cos\delta + \frac{5dq}{R^2}$