How to escape from singularities in Table 6 through 9 of Okada (1992)

[1]　When $q = 0$

This situation occurs at the point on the plane (i) in the Fig.5 of Okada (1992).

In this case, $\Theta = \tan^{-1}\frac{\xi\eta}{qR}$ in Table 6 becomes singular.

This term comes from the integration, $J = \int \frac{q}{R(R+\xi)} d\eta$ where $R^2 = \xi^2 + \eta^2 + q^2$

Since $\frac{1}{R(R+\xi)} = \frac{1}{R^2-\xi^2} - \frac{\xi}{R(R^2-\xi^2)}$ and $\left.\frac{1}{R^2-\xi^2}\right|_{\xi=\xi_1}^{\xi=\xi_2} = 0$,

$$J = \int \frac{q}{R(R+\xi)} d\eta = -\xi q \int \frac{1}{R(R^2-\xi^2)} d\eta = -\xi q \int \frac{d\eta}{(\eta^2+q^2)\sqrt{\eta^2+q^2+\xi^2}} = \begin{cases} -\tan^{-1}\frac{\xi\eta}{qR} & \text{for } q \neq 0 \\ \frac{qR}{\xi\eta} & \text{for } q = 0 \end{cases} = \begin{cases} \Theta & \text{for } q \neq 0 \\ 0 & \text{for } q = 0 \end{cases}$$

Here we have used the mathematical formula

$$\int \frac{dx}{(x^2+a^2)\sqrt{x^2+a^2+b^2}} = \begin{cases} \frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\frac{x}{\sqrt{x^2+a^2+b^2}}\right) & \text{for } a \neq 0, b \neq 0 \\ -\frac{\sqrt{x^2+a^2}}{a^2 x} & \text{for } a \neq 0, b = 0 \\ -\frac{\sqrt{x^2+b^2}}{b^2 x} & \text{for } a = 0, b \neq 0 \\ -\frac{1}{2x^2} & \text{for } a = 0, b = 0 \end{cases}$$

Therefore, when $q = 0$, we should set Θ in Table 6 to be zero.

[Column 1]　The role of Θ and the meaning to set it zero when $q = 0$

When $\xi\eta > 0$, the term $\Theta = \tan^{-1}\frac{\xi\eta}{qR}$ converges to $+\pi/2$ when we approach to the front side of the fault surface ($q \to 0^+$), while it converges to $-\pi/2$ at the rear side ($q \to 0^-$) and vice versa for $\xi\eta < 0$. This term gives the discontinuity across the fault when both of ξ, η change their signs in the Chinnery's operation. Such a case occurs only when the point lies on the fault surface itself.

In Table 6, Θ appears in the 1st, 2nd, and 3rd component for strike-slip, dip-slip, and tensile faults. They are parallel to each direction of dislocations. Since the terms other than Θ are continuous across the fault surface, to set $\Theta = 0$ on the fault surface ($q = 0$) gives the average of the displacements at both sides of the fault surface calculated from $\Theta = +\pi/2$ and $\Theta = -\pi/2$, while all the other components are kept to be continuous across the fault surface.

-π/2+π/2+π/2-π/2	π/2+π/2-π/2-π/2	π/2-π/2-π/2+π/2
-π/2+π/2-π/2+π/2	π/2+π/2+π/2+π/2	π/2-π/2+π/2-π/2
π/2-π/2-π/2+π/2	-π/2-π/2+π/2+π/2	-π/2+π/2+π/2-π/2

[2] When $\xi = 0$

This situation occurs at the point on the plane (ii), i.e. $x = 0$ or $x = L$ in the Fig.5 of Okada (1992).

In this case, arctan term of $I_4 = \frac{\sin\delta}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{2}{\cos^2\delta}\tan^{-1}\frac{\eta(X+q\cos\delta)+X(R+X)\sin\delta}{\xi(R+X)\cos\delta}$ in Table 6 becomes singular.

This term comes from the integration of $I_5^0 = \frac{1}{R(R+d)} - x^2\frac{2R+d}{R^3(R+d)^2}$ in Table 2.

After substitution $\begin{cases} x \to \xi \\ y \to \tilde{y} = \eta\cos\delta + q\sin\delta \\ d \to \tilde{d} = \eta\sin\delta - q\cos\delta \\ p \to \eta \\ R^2 \to \xi^2 + \eta^2 + q^2 = X^2 + \eta^2 \end{cases}$, we need integration $\int_x^{x-L} d\xi \int_p^{p-W} d\eta$.

At first, the integration by ξ becomes $\int I_5^0 d\xi = \int \left[\frac{1}{R(R+\tilde{d})} - \xi^2\frac{2R+\tilde{d}}{R^3(R+\tilde{d})^2}\right] d\xi = \frac{\xi}{R(R+\tilde{d})}$

Next, to perform integration by η, we change integral variable $\eta \to t = \frac{R-X}{\eta} = \frac{\eta}{R+X}$.

Using $R = \frac{1+t^2}{1-t^2}X,\ \eta = \frac{2t}{1-t^2}X,\ d\eta = \frac{2(1+t^2)}{(1-t^2)^2}Xdt,$

$$\iint I_5^0 d\xi = \int \frac{\xi}{R(R+\tilde{d})}d\eta = \int \frac{\xi}{\frac{1+t^2}{1-t^2}X\left(\frac{1+t^2}{1-t^2}X + \frac{2t\sin\delta}{1-t^2}X - q\cos\delta\right)}\frac{2(1+t^2)}{(1-t^2)^2}Xdt$$

$$= \int \frac{2\xi}{(X+q\cos\delta)t^2 + (2X\sin\delta)t + (X-q\cos\delta)}dt \quad \left[= 2\xi\int\frac{1}{at^2+bt+c}dt\,,\quad b^2-4ac = -4\xi^2\cos^2\delta\right]$$

$$= \begin{cases} \frac{2}{\cos\delta}\tan^{-1}\frac{(X+q\cos\delta)t + X\sin\delta}{\xi\cos\delta} = \frac{2}{\cos\delta}\tan^{-1}\frac{\eta(X+q\cos\delta)+X(R+X)\sin\delta}{\xi(R+X)\cos\delta} & \text{for } \xi\cos\delta \neq 0 \\ -\frac{2\xi}{(X+q\cos\delta)t + X\sin\delta} = -\frac{2\xi(R+X)}{\eta(X+q\cos\delta)+X(R+X)\sin\delta} & \text{for } \xi\cos\delta = 0 \end{cases}$$

Here we have applied the following mathematical formula for $b^2 < 4ac$ case.

$$\int\frac{dx}{ax^2+bx+c} = \begin{cases} \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| & \text{for } b^2 > 4ac \\ \frac{1}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } b^2 < 4ac \\ -\frac{2}{2ax+b} & \text{for } b^2 = 4ac \end{cases}$$

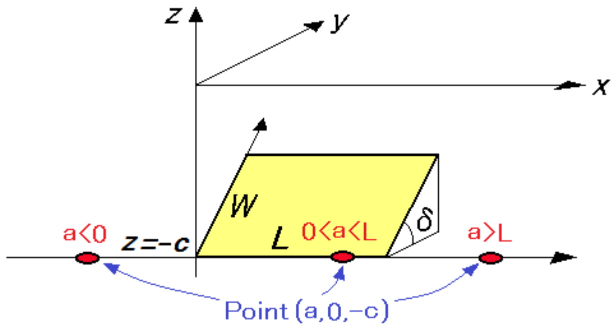
Therefore, when $\xi = 0$, we should set I_4 in Table 6 to be zero.

[3] When $R + \xi = 0$

This situation occurs at the point on the line (iii) in the Fig.5 of Okada (1992). In this case, $\ln(R+\xi)$ and the terms including $R+\xi$ in their denominators in Tables 6 to 9 become singular.

The condition $R+\xi=0$ happens when $\xi<0$ and $\eta=q=0$ in the terms related to $f_i^A(\xi,\eta,-z)$ in Tables 6 through 9, namely the contribution from the real fault in an infinite medium. Since the other terms, $f_i^A(\xi,\eta,z)$, $f_i^B(\xi,\eta,z)$, $f_i^C(\xi,\eta,z)$ are the contributions from the image fault, R, the distance from the corner of the image fault, is always greater than $|\xi|$.

In Tables 6 through 9, the elements including $R+\xi$ in their argument or denominator in $f_i^A(\xi,\eta,-z)$ related terms are $\ln(R+\xi)$, X_{11}, X_{32}, among which X_{11} appear as the combinations of ηX_{11}, qX_{11}, $\tilde{y}X_{11}$, $\tilde{d}X_{11}$, ηqX_{11}, q^2X_{11}, and X_{32} as the combinations of $\eta q\tilde{y}X_{32}$, $\eta q\tilde{d}X_{32}$, $q^2\tilde{y}X_{32}$, $q^2\tilde{d}X_{32}$.



Here, let us consider the behavior of these functions in the vicinity of the point $(a,0,-c)$.

After putting $\begin{cases} x=a \\ y=\varepsilon \\ z=-c+\varepsilon' \end{cases}$, $\begin{cases} d=c+z=\varepsilon' \quad \text{(because } -z \text{ is inserted)} \\ p=y\cos\delta+d\sin\delta=\varepsilon\cos\delta+\varepsilon'\sin\delta=\varepsilon_1 \\ q=y\sin\delta-d\cos\delta=\varepsilon\sin\delta-\varepsilon'\cos\delta=\varepsilon_2 \end{cases}$

We must evaluate $f(\xi,\eta)\| = f(x,p)-f(x-L,p)-f(x,p-W)+f(x-L,p-W)$

$$= f(a,\varepsilon_1)-f(a-L,\varepsilon_1)-f(a,\varepsilon_1-W)+f(a-L,\varepsilon_1-W)$$

(a) $f(\xi,\eta)=\ln(R+\xi)$

$$\begin{cases} f(a,\varepsilon_1)=\ln\left(\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+a\right)=\ln\left[|a|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{a^2}\right)^{1/2}+a\right]=\ln\left(|a|+a+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}\right) \\ f(a-L,\varepsilon_1)=\ln\left(|a-L|+(a-L)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}\right) \\ f(a,\varepsilon_1-W)=\ln\left(\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a\right) \\ f(a-L,\varepsilon_1-W)=\ln\left(\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+(a-L)\right) \end{cases}$$

When $\varepsilon_1\to 0$, $\varepsilon_2\to 0$, the 3rd and 4th terms become definite without any singularity. The 1st and 2nd terms of $\ln(R+\xi)\|$ becomes as follows.

$$\ln\left(|a|+a+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}\right)-\ln\left(|a-L|+(a-L)+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}\right)$$

$$=\begin{cases} \ln\left(2a+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a}\right)-\ln\left(2(a-L)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(a-L)}\right) \to \ln\dfrac{a}{a-L} & \text{for } a>L \\ \ln\left(2a+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a}\right)-\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(L-a)} \to \infty & \text{for } 0<a<L \\ \ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{-2a}-\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(L-a)}=\ln\dfrac{L-a}{-a} & \text{for } a<0 \end{cases}$$

When $0<a<L$, the point lies on the edge and the value of $\ln(R+\xi)$ becomes infinite (singular). When $a<0$, the value of $\ln(R+\xi)$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Here, since $\ln(R+\xi) = \ln(R^2-\xi^2) - \ln(R-\xi)$ and $\ln(R^2-\xi^2)|_{\xi=x}^{\xi=x-L} = 0$, we can use $-\ln(R-\xi)$ instead of $\ln(R+\xi)$.

For $f(\xi,\eta) = -\ln(R-\xi)$

$$\begin{cases} f(a,\varepsilon_1) = -\ln\left(\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}-a\right) = -\ln\left[|a|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{a^2}\right)^{1/2}-a\right] = -\ln\left(|a|-a+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}\right) \\ f(a-L,\varepsilon_1) = -\ln\left(|a-L|-(a-L)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}\right) \\ f(a,\varepsilon_1-W) = -\ln\left(\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}-a\right) \\ f(a-L,\varepsilon_1-W) = -\ln\left(\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}-(a-L)\right) \end{cases}$$

When $\varepsilon_1 \to 0$, $\varepsilon_2 \to 0$, the 3rd and 4th terms become definite without any singularity.

The 1st and 2nd terms of $-\ln(R-\xi)\|$ becomes as follows.

$$-\ln\left(|a|-a+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}\right) + \ln\left(|a-L|-(a-L)+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}\right)$$

$$= \begin{cases} -\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a} + \ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(a-L)} = -\ln\dfrac{a-L}{a} & \text{for } a > L \\ -\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a} + \ln\left(2(L-a)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(L-a)}\right) \quad \to \quad \infty & \text{for } 0 < a < L \\ -\ln\left(-2a+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{-2a}\right) + \ln\left(2(L-a)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(L-a)}\right) \quad \to \quad -\ln\dfrac{-a}{L-a} & \text{for } a < 0 \end{cases}$$

When $0 < a < L$, the point lies on the edge and the value of $-\ln(R-\xi)$ becomes infinite (singular). When $a > L$, the value of $-\ln(R-\xi)$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R+\xi = 0$, it is appropriate to use $-\ln(R-\xi)$ instead of $\ln(R+\xi)$ to avoid the numerical singularity. When the point lies on the fault edge we should add a flag of singularity to the output.

(b) $f(\xi,\eta) = \eta X_{11} = \frac{\eta}{R(R+\xi)}$

$$\begin{cases} f(a,\varepsilon_1) = \dfrac{\varepsilon_1}{|a|\left(\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+a\right)} = \dfrac{\varepsilon_1}{|a|\left[|a|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a^2}\right)+a\right]} = \dfrac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a|(|a|+a)} \\ f(a-L,\varepsilon_1) = \dfrac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a-L|(|a-L|+a-L)} \\ f(a,\varepsilon_1-W) = \dfrac{\varepsilon_1-W}{\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a\right)} \\ f(a-L,\varepsilon_1-W) = \dfrac{\varepsilon_1-W}{\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a-L\right)} \end{cases}$$

When $\varepsilon_1 \to 0$, $\varepsilon_2 \to 0$, the 3rd and 4th terms become definite without any singularity.

The 1st and 2nd terms of $\eta X_{11}\|$ becomes as follows.

$$\frac{\varepsilon_1}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a|(|a|+a)}-\frac{\varepsilon_1}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a-L|(|a-L|+a-L)}$$

$$=\begin{cases} \frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4a^2}-\frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4(a-L)^2} \to 0 & \text{for } a>L \\ \frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4a^2}-\frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2} \to \infty & \text{for } 0<a<L \\ \frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2}-\frac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2}=0 & \text{for } a<0 \end{cases}$$

When $0<a<L$, the point lies on the edge and the value of ηX_{11} becomes infinite (singular). When $a>L$, the value of ηX_{11} itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R+\xi=0$, it is appropriate to set $X_{11}=\frac{1}{R(R+\xi)}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(c) $f(\xi,\eta)=qX_{11}=\frac{q}{R(R+\xi)}$

It can be applied the same discussion as (b) $f(\xi,\eta)=\eta X_{11}=\frac{\eta}{R(R+\xi)}$

(d) $f(\xi,\eta)=\tilde{y}X_{11}=\frac{\eta\cos\delta+q\sin\delta}{R(R+\xi)}$

It can be applied the same discussion as (b) $f(\xi,\eta)=\eta X_{11}=\frac{\eta}{R(R+\xi)}$

(e) $f(\xi,\eta)=\tilde{d}X_{11}=\frac{\eta\sin\delta-q\cos\delta}{R(R+\xi)}$

It can be applied the same discussion as (b) $f(\xi,\eta)=\eta X_{11}=\frac{\eta}{R(R+\xi)}$

(f) $f(\xi,\eta)=\eta qX_{11}=\frac{\eta q}{R(R+\xi)}$

$$\begin{cases} f(a,\varepsilon_1)=\frac{\varepsilon_1\varepsilon_2}{|a|\left(\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+a\right)}=\frac{\varepsilon_1\varepsilon_2}{|a|\left[|a|\left(1+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2a^2}\right)+a\right]}=\frac{\varepsilon_1\varepsilon_2}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a|(|a|+a)} \\ f(a-L,\varepsilon_1)=\frac{\varepsilon_1\varepsilon_2}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a-L|(|a-L|+a-L)} \\ f(a,\varepsilon_1-W)=\frac{(\varepsilon_1-W)\varepsilon_2}{\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a\right)} \\ f(a-L,\varepsilon_1-W)=\frac{(\varepsilon_1-W)\varepsilon_2}{\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a-L\right)} \end{cases}$$

Since the 3rd and 4th terms vanish when $\varepsilon_1\to 0,\ \varepsilon_2\to 0$, we can neglect them.

$$f(\xi,\eta)\| = \eta q X_{11}\| = \frac{\varepsilon_1\varepsilon_2}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a|(|a|+a)} - \frac{\varepsilon_1\varepsilon_2}{\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|a-L|(|a-L|+a-L)}$$

$$= \begin{cases} \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4a^2} - \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4(a-L)^2} \quad \to \quad 0 & \text{for } a > L \\ \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4a^2} - \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} \quad \to \quad \text{indefinite} & \text{for } 0 < a < L \\ \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} - \frac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} = 0 & \text{for } a < 0 \end{cases}$$

When $0 < a < L$, the point lies on the edge and the value of $\eta q X_{11}$ becomes indefinite (singular). When $a < 0$, the value of $\eta q X_{11}$ itself becomes indefinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R + \xi = 0$, it is appropriate to set $X_{11} = \frac{1}{R(R+\xi)}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(g) $f(\xi,\eta) = q^2 X_{11} = \frac{q^2}{R(R+\xi)}$

It can be applied the same discussion as (f) $f(\xi,\eta) = \eta q X_{11} = \frac{\eta q}{R(R+\xi)}$

(h) $f(\xi,\eta) = \eta q \tilde{y} X_{32} = \eta q(\eta\cos\delta + q\sin\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

$$\begin{cases} f(a,\varepsilon_1) = \varepsilon_1\varepsilon_2(\varepsilon_1\cos\delta+\varepsilon_2\sin\delta)\frac{2\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+a}{|a|^3\left(\sqrt{a^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+a\right)^2} = \varepsilon_1\varepsilon_2\varepsilon\frac{2|a|+a+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}}{|a|^3\left(|a|+a+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a|}\right)^2} \\ f(a-L,\varepsilon_1) = \varepsilon_1\varepsilon_2\varepsilon\frac{2|a-L|+a-L+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}}{|a-L|^3\left(|a-L|+a-L+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|a-L|}\right)^2} \\ f(a,\varepsilon_1-W) = (\varepsilon_1-W)\varepsilon_2(\varepsilon-W\cos\delta)\frac{2\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a}{\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}^3\left(\sqrt{a^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a\right)^2} \\ f(a-L,\varepsilon_1-W) = (\varepsilon_1-W)\varepsilon_2(\varepsilon-W\cos\delta)\frac{2\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a-L}{\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}^3\left(\sqrt{(a-L)^2+(W-\varepsilon_1)^2+{\varepsilon_2}^2}+a-L\right)^2} \end{cases}$$

Since the 3rd and 4th terms vanish when $\varepsilon_1 \to 0$, $\varepsilon_2 \to 0$, we can neglect them.

$$f(\xi,\eta)\| = \eta q\tilde{y}X_{32}\| = \varepsilon_1\varepsilon_2\varepsilon\frac{2a^2+a|a|+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}}{|a|^3\left(a^2+a|a|+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}\right)^2} - \varepsilon_1\varepsilon_2\varepsilon\frac{2(a-L)^2+(a-L)|a-L|+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}}{|a-L|^3\left((a-L)^2+(a-L)|a-L|+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}\right)^2}$$

$$= \begin{cases} \frac{3a^2\varepsilon_1\varepsilon_2\varepsilon}{4a^6} - \frac{3(a-L)^2\varepsilon_1\varepsilon_2\varepsilon}{4(a-L)^6} \quad \to \quad 0 & \text{for } a > L \\ \frac{3a^2\varepsilon_1\varepsilon_2\varepsilon}{4a^6} - \frac{4(L-a)^2\varepsilon_1\varepsilon_2\varepsilon}{(L-a)^2({\varepsilon_1}^2+{\varepsilon_2}^2)^2} \quad \to \quad \infty & \text{for } 0 < a < L \\ -\frac{4a^2\varepsilon_1\varepsilon_2\varepsilon}{a^2({\varepsilon_1}^2+{\varepsilon_2}^2)^2} - \frac{4(L-a)^2\varepsilon_1\varepsilon_2\varepsilon}{(L-a)^2({\varepsilon_1}^2+{\varepsilon_2}^2)^2} = 0 & \text{for } a < 0 \end{cases}$$

When $0 < a < L$, the point lies on the edge and the value of $\eta q \tilde{y} X_{32}$ becomes infinite (singular). When $a < 0$, the value of $\eta q \tilde{y} X_{32}$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R + \xi = 0$, it is appropriate to set $X_{32} = \frac{2R+\xi}{R^3(R+\xi)^2}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(i) $f(\xi,\eta) = \eta q \tilde{d} X_{32} = \eta q(\eta \sin\delta - q\cos\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

It can be applied the same discussion as (h) $f(\xi,\eta) = \eta q \tilde{y} X_{32} = \eta q(\eta\cos\delta + q\sin\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

(j) $f(\xi,\eta) = q^2 \tilde{y} X_{32} = q^2(\eta\cos\delta + q\cos\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

It can be applied the same discussion as (h) $f(\xi,\eta) = \eta q \tilde{y} X_{32} = \eta q(\eta\cos\delta + q\sin\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

(k) $f(\xi,\eta) = q^2 \tilde{d} X_{32} = q^2(\eta\sin\delta - q\cos\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

It can be applied the same discussion as (h) $f(\xi,\eta) = \eta q \tilde{y} X_{32} = \eta q(\eta\cos\delta + q\sin\delta)\frac{2R+\xi}{R^3(R+\xi)^2}$

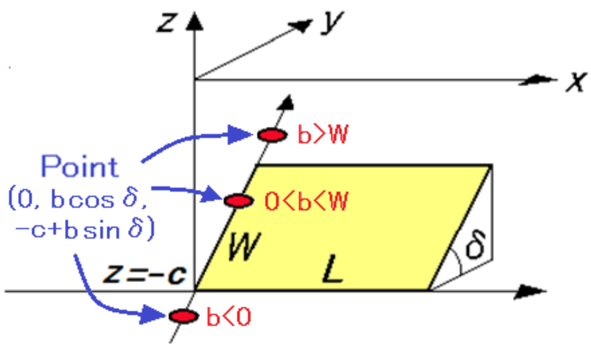
[4] When $R + \eta = 0$

This situation occurs at the point on the line (iv) in the Fig.5 of Okada (1992). In this case, $\ln(R+\eta)$ and the terms including $R+\eta$ in their denominators in Tables 6 to 9 become singular.

The condition $R + \eta = 0$ happens when $\eta < 0$ and $\xi = q = 0$ in the terms related to $f_i^A(\xi,\eta,-z)$ in Tables 6 through 9, namely the contribution from the real fault in an infinite medium. Since the other terms, $f_i^A(\xi,\eta,z)$, $f_i^B(\xi,\eta,z)$, $f_i^C(\xi,\eta,z)$ are the contributions from the image fault, R, the distance from the corner of the image fault, is always greater than $|\eta|$.

In Tables 6 through 9, the elements including $R+\eta$ in their argument or denominator in $f_i^A(\xi,\eta,-z)$ related terms are $\ln(R+\eta)$, Y_{11}, Y_{32}, among which Y_{11} appear as the combinations of ξY_{11}, qY_{11}, $\xi q Y_{11}$, and Y_{32}as the combinations of $\xi^2 q Y_{32}$, $\xi q^2 Y_{32}$, $\xi^3 Y_{32}$, $q^3 Y_{32}$.

Here, let us consider the behavior of these functions in the vicinity of the point $(0, b\cos\delta, -c + b\sin\delta)$.



After putting $\begin{cases} x = \varepsilon_1 \\ y = b\cos\delta + \varepsilon_2\ \sin\delta \\ z = -c + b\sin\delta - \varepsilon_2\cos\delta \end{cases}$, $\begin{cases} d = c + z = b\sin\delta - \varepsilon_2\cos\delta \\ p = y\cos\delta + d\sin\delta = b \\ q = y\sin\delta - d\cos\delta = \varepsilon_2 \end{cases}$

We must evaluate $f(\xi,\eta)\| = f(x,p) - f(x,p-W) - f(x-L,p) + f(x-L,p-W)$

$$= f(\varepsilon_1,b) - f(\varepsilon_1,b-W) - f(\varepsilon_1-L,b) + f(\varepsilon_1-L,b-W)$$

(a) $f(\xi,\eta) = \ln(R+\eta)$

$$\begin{cases} f(\varepsilon_1,b) = \ln\left(\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+b\right) = \ln\left[|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{b^2}\right)^{1/2}+b\right] = \ln\left(|b|+b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}\right) \\ f(\varepsilon_1,b-W) = \ln\left(|b-W|+(b-W)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}\right) \\ f(\varepsilon_1-L,b) = \ln\left(\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b\right) \\ f(\varepsilon_1-L,b-W) = \ln\left(\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+(b-W)\right) \end{cases}$$

When $\varepsilon_1 \to 0,\ \varepsilon_2 \to 0$, the 3rd and 4th terms become definite without any singularity.
The 1st and 2nd terms of $\ln(R+\eta)\|$ becomes as follows.

$$\ln\left(|b|+b+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}\right) - \ln\left(|b-W|+(b-W)+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}\right)$$

$$= \begin{cases} \ln\left(2b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b}\right) - \ln\left(2(b-W)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(b-W)}\right) \quad \to \quad \ln\dfrac{b}{b-W} & \text{for } b > W \\ \ln\left(2b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b}\right) - \ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(W-b)} \quad \to \quad \infty & \text{for } 0 < b < W \\ \ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{-2b} - \ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(W-b)} = \ln\dfrac{W-b}{-b} & \text{for } b < 0 \end{cases}$$

When $0 < b < W$, the point lies on the edge and the value of $\ln(R+\eta)$ becomes infinite (singular). When $b < 0$, the value of $\ln(R+\eta)$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Here, since $\ln(R+\eta) = \ln(R^2-\eta^2) - \ln(R-\eta)$ and $\ln(R^2-\eta^2)|_{\eta=p}^{\eta=p-W} = 0$, we can use $-\ln(R-\eta)$ instead of $\ln(R+\eta)$.

For $f(\xi,\eta) = -\ln(R-\eta)$

$$\begin{cases} f(\varepsilon_1,b) = -\ln\left(\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}-b\right) = -\ln\left[|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{b^2}\right)^{1/2}-b\right] = -\ln\left(|b|-b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}\right) \\ f(\varepsilon_1,b-W) = -\ln\left(|b-W|-(b-W)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}\right) \\ f(\varepsilon_1-L,b) = -\ln\left(\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}-b\right) \\ f(\varepsilon_1-L,b-W) = -\ln\left(\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}-(b-W)\right) \end{cases}$$

When $\varepsilon_1 \to 0,\ \varepsilon_2 \to 0$, the 3rd and 4th terms become definite without any singularity.
The 1st and 2nd terms of $-\ln(R-\eta)\|$ becomes as follows.

$$-\ln\left(|b|-b+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}\right)+\ln\left(|b-W|-(b-W)+\frac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}\right)$$

$$=\begin{cases} -\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b}+\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(b-W)}=\ln\dfrac{b}{b-W} & \text{for } b>W \\ -\ln\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b}+\ln\left(2(W-b)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(W-b)}\right) \quad\to\quad \infty & \text{for } 0<b<W \\ -\ln\left(-2b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{-2b}\right)+\ln\left(2(W-b)+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2(W-b)}\right) \quad\to\quad \ln\dfrac{-b}{W-b} & \text{for } b<0 \end{cases}$$

When $0<b<W$, the point lies on the edge and the value of $-\ln(R-\eta)$ becomes infinite (singular). When $b>W$, the value of $-\ln(R-\eta)$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R+\eta=0$, it is appropriate to use $-\ln(R-\eta)$ instead of $\ln(R+\eta)$ to avoid the numerical singularity. When the point lies on the fault edge we should add a flag of singularity to the output.

(b) $f(\xi,\eta)=\xi Y_{11}=\frac{\xi}{R(R+\eta)}$

$$\begin{cases} f(\varepsilon_1,b)=\dfrac{\varepsilon_1}{|b|\left(\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+b\right)}=\dfrac{\varepsilon_1}{|b|\left[|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b^2}\right)+b\right]}=\dfrac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b|(|b|+b)} \\ f(\varepsilon_1,b-W)=\dfrac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b-W|(|b-W|+b-W)} \\ f(\varepsilon_1-L,b)=\dfrac{\varepsilon_1-L}{\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b\right)} \\ f(\varepsilon_1-L,b-W)=\dfrac{\varepsilon_1-L}{\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b-W\right)} \end{cases}$$

When $\varepsilon_1\to 0$, $\varepsilon_2\to 0$, the 3rd and 4th terms become definite without any singularity.
The 1st and 2nd terms of $\xi Y_{11}\|$ becomes as follows.

$$\frac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b|(|b|+b)}-\frac{\varepsilon_1}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b-W|(|b-W|+b-W)}$$

$$=\begin{cases} \dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4b^2}-\dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4(b-W)^2} \quad\to\quad 0 & \text{for } b>W \\ \dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2+4b^2}-\dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2} \quad\to\quad \infty & \text{for } 0<b<W \\ \dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2}-\dfrac{2\varepsilon_1}{{\varepsilon_1}^2+{\varepsilon_2}^2}=0 & \text{for } b<0 \end{cases}$$

When $0<b<W$, the point lies on the edge and the value of ξY_{11} becomes infinite (singular). When $b<0$, the value of ξY_{11} itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R+\eta=0$, it is appropriate to set $Y_{11}=\frac{1}{R(R+\eta)}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(c) $f(\xi,\eta) = qY_{11} = \frac{q}{R(R+\eta)}$

It can be applied the same discussion as (b) $f(\xi,\eta) = \xi Y_{11} = \frac{\xi}{R(R+\eta)}$

(d) $f(\xi,\eta) = \xi q Y_{11} = \frac{\xi q}{R(R+\eta)}$

$$\begin{cases} f(\varepsilon_1, b) = \dfrac{\varepsilon_1\varepsilon_2}{|b|\left(\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+b\right)} = \dfrac{\varepsilon_1\varepsilon_2}{|b|\left[|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b^2}\right)+b\right]} = \dfrac{\varepsilon_1\varepsilon_2}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b|(|b|+b)} \\ f(\varepsilon_1, b-W) = \dfrac{\varepsilon_1\varepsilon_2}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b-W|(|b-W|+b-W)} \\ f(\varepsilon_1-L, b) = \dfrac{(\varepsilon_1-L)\varepsilon_2}{\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b\right)} \\ f(\varepsilon_1-L, b-W) = \dfrac{(\varepsilon_1-L)\varepsilon_2}{\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}\left(\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b-W\right)} \end{cases}$$

Since the 3rd and 4th terms vanish when $\varepsilon_1 \to 0,\ \varepsilon_2 \to 0$, we can neglect them.

$$f(\xi,\eta)\| = \xi q Y_{11}\| = \frac{\varepsilon_1\varepsilon_2}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b|(|b|+b)} - \frac{\varepsilon_1\varepsilon_2}{\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2}+|b-W|(|b-W|+b-W)}$$

$$= \begin{cases} \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4b^2} - \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4(b-W)^2} \quad \to \quad 0 & \text{for } b > W \\ \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2+4b^2} - \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} \quad \to \quad \text{indefinite} & \text{for } 0 < b < W \\ \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} - \dfrac{2\varepsilon_1\varepsilon_2}{{\varepsilon_1}^2+{\varepsilon_2}^2} = 0 & \text{for } b < 0 \end{cases}$$

When $0 < b < W$, the point lies on the edge and the value of $\xi q Y_{11}$ becomes indefinite (singular). When $b < 0$, the value of $\xi q Y_{11}$ itself becomes indefinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R+\eta = 0$, it is appropriate to set $Y_{11} = \frac{1}{R(R+\eta)}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(e) $f(\xi,\eta) = \xi^2 q Y_{32} = \xi^2 q \frac{2R+\eta}{R(R+\eta)}$

$$\begin{cases} f(\varepsilon_1, b) = {\varepsilon_1}^2\varepsilon_2 \dfrac{2\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+b}{|b|^3\left(\sqrt{b^2+{\varepsilon_1}^2+{\varepsilon_2}^2}+b\right)^2} = {\varepsilon_1}^2\varepsilon_2 \dfrac{2|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b^2}\right)+b}{|b|^3\left[|b|\left(1+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2b^2}\right)+b\right]^2} = {\varepsilon_1}^2\varepsilon_2 \dfrac{2|b|+b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}}{|b|^3\left(|b|+b+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b|}\right)^2} \\ f(\varepsilon_1, b-W) = {\varepsilon_1}^2\varepsilon_2 \dfrac{2|b-W|+b-W+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}}{|b-W|^3\left(|b-W|+b-W+\dfrac{{\varepsilon_1}^2+{\varepsilon_2}^2}{2|b-W|}\right)^2} \\ f(\varepsilon_1-L, b) = (L-\varepsilon_1)^2\varepsilon_2 \dfrac{2\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b}{\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}^3\left(\sqrt{b^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b\right)^2} \\ f(\varepsilon_1-L, b-W) = (L-\varepsilon_1)^2\varepsilon_2 \dfrac{2\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b-W}{\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}^3\left(\sqrt{(b-W)^2+(L-\varepsilon_1)^2+{\varepsilon_2}^2}+b-W\right)^2} \end{cases}$$

Since the 3rd and 4th terms vanish when $\varepsilon_1 \to 0,\ \varepsilon_2 \to 0$, we can neglect them.

$$f(\xi,\eta)\| = \xi^2 qY_{32}\| = {\varepsilon_1}^2\varepsilon_2 \frac{2b^2 + b|b| + \frac{{\varepsilon_1}^2 + {\varepsilon_2}^2}{2}}{b^2\left(b^2 + b|b| + \frac{{\varepsilon_1}^2 + {\varepsilon_2}^2}{2}\right)^2} - {\varepsilon_1}^2\varepsilon_2 \frac{2(b-W)^2 + (b-W)|b-W| + \frac{{\varepsilon_1}^2 + {\varepsilon_2}^2}{2}}{(b-W)^2\left((b-W)^2 + (b-W)|b-W| + \frac{{\varepsilon_1}^2 + {\varepsilon_2}^2}{2}\right)^2}$$

$$= \begin{cases} \dfrac{3b^2{\varepsilon_1}^2\varepsilon_2}{4b^6} - \dfrac{3(b-W)^2{\varepsilon_1}^2\varepsilon_2}{4(b-W)^6} \quad \to \quad 0 & \text{for } b > W \\ \dfrac{3b^2{\varepsilon_1}^2\varepsilon_2}{4b^6} - \dfrac{4(b-W)^2{\varepsilon_1}^2\varepsilon_2}{(b-W)^2({\varepsilon_1}^2 + {\varepsilon_2}^2)^2} \quad \to \quad \infty & \text{for } 0 < b < W \\ -\dfrac{4b^2{\varepsilon_1}^2\varepsilon_2}{b^2({\varepsilon_1}^2 + {\varepsilon_2}^2)^2} - \dfrac{4(b-W)^2{\varepsilon_1}^2\varepsilon_2}{(b-W)^2({\varepsilon_1}^2 + {\varepsilon_2}^2)^2} = 0 & \text{for } b < 0 \end{cases}$$

When $0 < b < W$, the point lies on the edge and the value of $\xi^2 qY_{32}$ becomes infinite (singular). When $b < 0$, the value of $\xi^2 qY_{32}$ itself becomes infinite but cancelled out each other by Chinnery's operation.

Therefore, in the case of $R + \eta = 0$, it is appropriate to set $Y_{32} = \frac{2R+\eta}{R^3(R+\eta)^2}$ to be zero to avoid numerical singularity, ZERO-DIV. When the point lies on the fault edge we should add a flag of singularity to the output.

(f) $f(\xi,\eta) = \xi q^2 Y_{32} = \xi q^2 \frac{2R+\eta}{R(R+\eta)}$

It can be applied the same discussion as (e) $f(\xi,\eta) = \xi^2 qY_{32} = \xi^2 q \frac{2R+\eta}{R(R+\eta)}$

(g) $f(\xi,\eta) = \xi^3 Y_{32} = \xi^3 \frac{2R+\eta}{R(R+\eta)}$

It can be applied the same discussion as (e) $f(\xi,\eta) = \xi^2 qY_{32} = \xi^2 q \frac{2R+\eta}{R(R+\eta)}$

(h) $f(\xi,\eta) = q^3 Y_{32} = q^3 \frac{2R+\eta}{R(R+\eta)}$

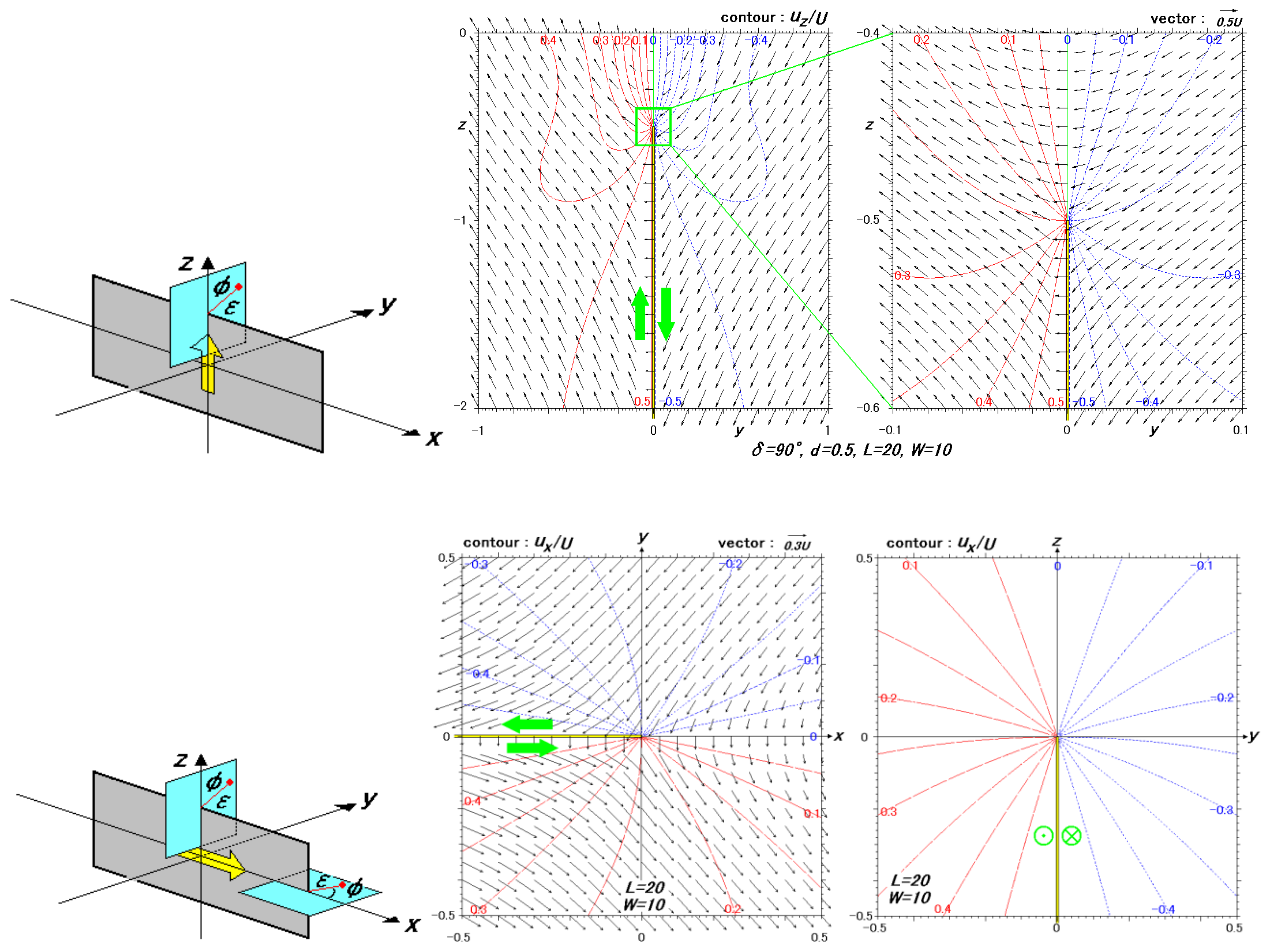
It can be applied the same discussion as (e) $f(\xi,\eta) = \xi^2 qY_{32} = \xi^2 q \frac{2R+\eta}{R(R+\eta)}$

[Column 2]   Singularities on the fault edge

For the case of $R + \xi = 0$ or $R + \eta = 0$, we can avoid mathematical singularity of ZERO-DIV by putting X_{11}, X_{32}, Y_{11}, Y_{32} to be zero. However, for the point on the fault edge, we cannot escape from the essential singularity.

The points on the fault surface have singularity in the sense of a double-valued. A certain component of the displacement has alternate values depending whether we approach to the fault surface from the front side or from the rear side. This is just the dislocation itself.

On the other hand, the points on the fault edge have more high level singularity in the sense of a multi-valued. The displacement at these points has different value depending from which direction we approach to the point. The next figure shows an example of such a situation.



(top) Vertical displacement field around a Z-edge of a vertical dip-slip fault.

(bottom) Horizontal displacement field around a X- and Z-edge of a vertical strike-slip fault.

[cited from Okada,Y. (2003) Paradox on vertical displacement due to a fault model, *J. Geod. Soc. Japan*, **49**, 99-119 (in Japanese with English abstract)]