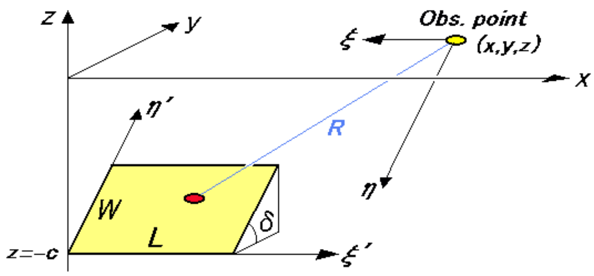
Derivation of Table 6 in Okada (1992)

[I] Integration for a finite rectangular source

Point source solutions given in Tables 2 through 5 have the form of $u^0(\mathrm{x,y,z}) = \frac{Mo}{2\pi\mu}[\cdots\cdots\cdots]$.
For a finite fault with a dislocation U, we can replace M_0 to $\mu U \iint_\Sigma [\cdots\cdots\cdots]d\Sigma$ using the concept of body force equivalents. This operation yields the finite fault solution in the form of $u(x,y,z) = \frac{U}{2\pi}\iint_\Sigma [\cdots\cdots\cdots]d\Sigma$.

To get finite fault solutions, we need double integration with $(\xi',\ \eta')$ after replacing the location of point source from $(0,0,-c)$ to $(\xi',\ \eta'\cos\delta,\ -c+\eta'\sin\delta)$.

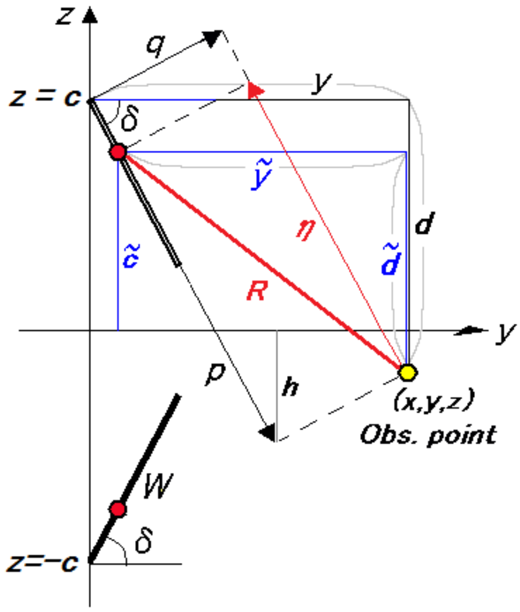


Namely, after changing

$$\begin{cases} x \ \to\ x-\xi' \\ y \ \to\ y-\eta'\cos\delta \\ c \ \to\ c-\eta'\sin\delta \end{cases}$$

in the point source solution, we need an operation

$$\int_0^L d\xi' \int_0^W d\eta'$$



Here, for the sake of convenience, we change the integration variables from $(\xi',\ \eta')$ to $\begin{cases}\xi = x-\xi' \\ \eta = p-\eta'\end{cases}$

Then, we should change the variables in the point source solution to

$$\begin{cases} x \ \to\ \xi \\ y \ \to\ \tilde{y} = y-(p-\eta)\cos\delta = \eta\cos\delta + q\sin\delta \\ d \ \to\ \tilde{d} = \ d-(p-\eta)\sin\delta = \eta\sin\delta - q\cos\delta \\ c \ \to\ \tilde{c} = \ \tilde{d}+z = \eta\sin\delta - h \qquad (h = q\cos\delta - z) \end{cases}$$

$$R^2 = \xi^2+\eta^2+q^2 = \xi^2+\tilde{y}^2+\tilde{d}^2$$

and perform the integration

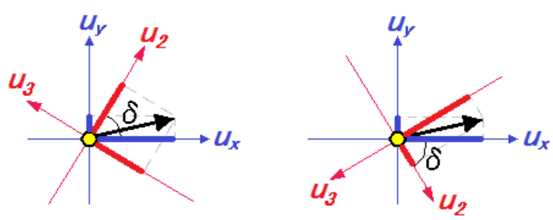
$$\int_x^{x-L} d\xi \int_p^{p-W} d\eta$$

where $\begin{cases} p = y\cos\delta + d\sin\delta \\ q = y\sin\delta - d\cos\delta \end{cases}$, $\quad d = c-z$

In the following, for the sake of simplicity, we will treat the displacement $(u_1,\ u_2,\ u_3)$ instead of $(u_x,\ u_y,\ u_z)$

For *A*- and *B*-parts of the displacement $\begin{cases} u_1 = \ \ u_x \\ u_2 = \ \ u_y\cos\delta + u_z\sin\delta \\ u_3 = -u_y\sin\delta + u_z\cos\delta \end{cases}$

and for the *C*-part of the displacement $\begin{cases} u_1 = \ \ u_x \\ u_2 = \ \ u_y\cos\delta - u_z\sin\delta \\ u_3 = -u_y\sin\delta - u_z\cos\delta \end{cases}$



for parts A and B for part C

The former u_2 corresponds to the displacement parallel to up-dip direction of the real fault, while the latter u_2 corresponds to that of the imaginary fault.

(1) Strike slip

Displacement due to a point strike-slip at $(0,0,-c)$ are given in Table 2 as follows.

$$u_A^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_x=\ \ \frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3x^2q}{R^5} \\ u_y=\ \ \frac{1-\alpha}{2}\frac{x}{R^3}\sin\delta+\frac{\alpha}{2}\frac{3xyq}{R^5} \\ u_z=-\frac{1-\alpha}{2}\frac{x}{R^3}\cos\delta+\frac{\alpha}{2}\frac{3xdq}{R^5} \end{array}\right)\qquad u_B^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_x=-\frac{3x^2q}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin\delta \\ u_y=-\frac{3xyq}{R^5}-\frac{1-\alpha}{\alpha}I_2^0\sin\delta \\ u_z=-\frac{3xdq}{R^5}-\frac{1-\alpha}{\alpha}I_4^0\sin\delta \end{array}\right)$$

$$u_C^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_x=-(1-\alpha)\frac{A_3}{R^3}\cos\delta+\alpha\frac{3cq}{R^5}A_5 \\ u_y=\ \ (1-\alpha)\frac{3xy}{R^5}\cos\delta+\alpha\frac{3cx}{R^5}\left(\sin\delta-\frac{5yq}{R^2}\right) \\ u_z=-(1-\alpha)\frac{3xy}{R^5}\sin\delta+\alpha\frac{3cx}{R^5}\left(\cos\delta+\frac{5dq}{R^2}\right)-\frac{3xq}{R^5} \end{array}\right)$$

$$I_1^0=y\left[\frac{1}{R(R+d)^2}-x^2\frac{3R+d}{R^3(R+d)^3}\right]$$
$$I_2^0=x\left[\frac{1}{R(R+d)^2}-y^2\frac{3R+d}{R^3(R+d)^3}\right]$$
$$I_4^0=-xy\frac{2R+d}{R^3(R+d)^2}$$

where, $d=c-z\,,\ q=y\sin\delta-d\cos\delta\,,\ R^2=x^2+y^2+d^2$

Here, for the sake of simplicity, the term $-\frac{3cxq}{R^5}$ in the z-component of u_B^o was restored to $-\frac{3xdq}{R^5}$ and the term $-\frac{3xq}{R^5}$ was added to the z-component of u_C^o (see "Derivation of Table 2").

If we convert the displacement (u_x, u_y, u_z) to (u_1, u_2, u_3) ,

$$u_A^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_1=\ \ \frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3x^2q}{R^5} \\ u_2=\ \ \frac{\alpha}{2}\frac{3xpq}{R^5} \\ u_3=-\frac{1-\alpha}{2}\frac{x}{R^3}-\frac{\alpha}{2}\frac{3xq^2}{R^5} \end{array}\right)\qquad u_B^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_1=-\frac{3x^2q}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin\delta \\ u_2=-\frac{3xpq}{R^5}-\frac{1-\alpha}{\alpha}(I_2^0\cos\delta+I_4^0\sin\delta)\sin\delta \\ u_3=\ \ \frac{3xq^2}{R^5}+\frac{1-\alpha}{\alpha}(I_2^0\sin\delta-I_4^0\cos\delta)\sin\delta \end{array}\right)$$

$$u_C^o=\frac{Mo}{2\pi\mu}\left(\begin{array}{l} u_1=-(1-\alpha)\frac{A_3}{R^3}\cos\delta+\alpha\frac{3cq}{R^5}A_5 \\ u_2=\ \ (1-\alpha)\frac{3xy}{R^5}-\alpha\frac{15cxpq}{R^7}+\frac{3xq}{R^5}\sin\delta \\ u_3=\ \ -\alpha\frac{3cx}{R^5}\left(1-\frac{5q^2}{R^2}\right)+\frac{3xq}{R^5}\cos\delta \end{array}\right)$$

For the integration, we substitute

$$\left\{\begin{array}{l} x\ \to\ \xi \\ y\ \to\ \tilde{y}=\eta\cos\delta+q\sin\delta \\ d\ \to\ \tilde{d}=\eta\sin\delta-q\cos\delta \\ c\ \to\ \tilde{c}=\tilde{d}+z=\eta\sin\delta-h \\ p\ \to\ \eta \\ q\ \to\ q \end{array}\right.\qquad \begin{array}{l} R^2=\xi^2+\eta^2+q^2=\xi^2+\tilde{y}^2+\tilde{d}^2 \\ h=q\cos\delta-z \end{array}$$

So, integrand becomes

$$u_A^o=\left(\begin{array}{l} u_1=\ \ \frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3\xi^2q}{R^5} \\ u_2=\ \ \frac{\alpha}{2}\frac{3\xi\eta q}{R^5} \\ u_3=-\frac{1-\alpha}{2}\frac{\xi}{R^3}-\frac{\alpha}{2}\frac{3\xi q^2}{R^5} \end{array}\right)\qquad u_B^o=\left(\begin{array}{l} u_1=-\frac{3\xi^2q}{R^5}-\frac{1-\alpha}{\alpha}I_1^0\sin\delta \\ u_2=-\frac{3\xi\eta q}{R^5}-\frac{1-\alpha}{\alpha}(I_2^0\cos\delta+I_4^0\sin\delta)\sin\delta \\ u_3=\ \ \frac{3\xi q^2}{R^5}+\frac{1-\alpha}{\alpha}(I_2^0\sin\delta-I_4^0\cos\delta)\sin\delta \end{array}\right)$$

$$u_C^o=\left(\begin{array}{l} u_1=-(1-\alpha)\left(\frac{1}{R^3}-\frac{3\xi^2}{R^5}\right)\cos\delta+\alpha q(\eta\sin\delta-h)\left(\frac{3}{R^5}-\frac{15\xi^2}{R^7}\right) \\ u_2=\ \ (1-\alpha)(\eta\cos\delta+q\sin\delta)\frac{3\xi}{R^5}-\alpha q(\eta\sin\delta-h)\frac{15\xi\eta}{R^7}+\frac{3\xi q}{R^5}\sin\delta \\ u_3=\ \ -\alpha(\eta\sin\delta-h)\left(\frac{3\xi}{R^5}-\frac{15\xi q^2}{R^7}\right)+\frac{3\xi q}{R^5}\cos\delta \end{array}\right)$$

$$I_1^0=\tilde{y}\left[\frac{1}{R(R+\tilde{d})^2}-\xi^2\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right]$$
$$I_2^0=\xi\left[\frac{1}{R(R+\tilde{d})^2}-\tilde{y}^2\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right]$$
$$I_4^0=-\xi\tilde{y}\frac{2R+\tilde{d}}{R^3(R+\tilde{d})^2}$$

At first, let us integrate with ξ (refer Appendix : Table of Integration)

$$\int u_A^o\,d\xi=\left(\begin{array}{l} u_1=-\frac{q}{2}X_{11}-\frac{\alpha}{2}\frac{\xi q}{R^3} \\ u_2=\ \ -\frac{\alpha}{2}\frac{\eta q}{R^3} \\ u_3=\ \ \frac{1-\alpha}{2}\frac{1}{R}+\frac{\alpha}{2}\frac{q^2}{R^3} \end{array}\right)\qquad \int u_B^o d\xi=\left(\begin{array}{l} u_1=\frac{\xi q}{R^3}+qX_{11}-\frac{1-\alpha}{\alpha}\int I_1^0 d\xi\,\sin\delta \\ u_2=\frac{\eta q}{R^3}-\frac{1-\alpha}{\alpha}\int(I_2^0\cos\delta+I_4^0\sin\delta)d\xi\sin\delta \\ u_3=-\frac{q^2}{R^3}+\frac{1-\alpha}{\alpha}\int(I_2^0\sin\delta-I_4^0\cos\delta)d\xi\sin\delta \end{array}\right)$$

$$\int u_C^o\,d\xi = \begin{pmatrix} u_1 = -(1-\alpha)\dfrac{\xi}{R^3}\cos\delta & +3\alpha\xi q\dfrac{\eta\sin\delta - h}{R^5} \\ u_2 = -(1-\alpha)\dfrac{\eta\cos\delta + q\sin\delta}{R^3} & +3\alpha\eta q\dfrac{\eta\sin\delta - h}{R^5} - \dfrac{q}{R^3}\sin\delta \\ u_3 = & \alpha(\eta\sin\delta - h)\left(\dfrac{1}{R^3} - \dfrac{3q^2}{R^5}\right) - \dfrac{q}{R^3}\cos\delta \end{pmatrix} \qquad X_{11} = \frac{1}{R(R+\xi)}$$

Next, let us integrate with η (refer Appendix : Table of Integration)

$$\iint u_A^o d\xi d\eta = \begin{pmatrix} u_1 = \dfrac{\Theta}{2} & +\dfrac{\alpha}{2}\xi q Y_{11} \\ u_2 = & \dfrac{\alpha}{2}\dfrac{q}{R} \\ u_3 = \dfrac{1-\alpha}{2}\ln(R+\eta) & -\dfrac{\alpha}{2}q^2 Y_{11} \end{pmatrix} \qquad \iint u_B^o d\xi d\eta = \begin{pmatrix} u_1 = -\xi q Y_{11} - \Theta & -\dfrac{1-\alpha}{\alpha}\iint I_1^0 d\xi d\eta \sin\delta \\ u_2 = -\dfrac{q}{R} & +\dfrac{1-\alpha}{\alpha}\iint(-I_2^0\cos\delta - I_4^0\sin\delta)d\xi d\eta \sin\delta \\ u_3 = q^2 Y_{11} & -\dfrac{1-\alpha}{\alpha}\iint(-I_2^0\sin\delta + I_4^0\cos\delta)d\xi d\eta \sin\delta \end{pmatrix}$$

$$\iint u_C^o d\xi d\eta = \begin{pmatrix} u_1 = (1-\alpha)\xi Y_{11}\cos\delta & -\alpha\xi q\left(\dfrac{\sin\delta}{R^3} - hY_{32}\right) \\ u_2 = (1-\alpha)\left(\dfrac{\cos\delta}{R} + qY_{11}\sin\delta\right) & -\alpha q\left[\left(\dfrac{\eta}{R^3} + Y_{11}\right)\sin\delta - \dfrac{h}{R^3}\right] + q\,Y_{11}\sin\delta \\ u_3 = & -\alpha\left[\left(\dfrac{1}{R} - \dfrac{q^2}{R^3}\right)\sin\delta - h\,(Y_{11} - q^2 Y_{32})\right] + q\,Y_{11}\cos\delta \end{pmatrix}$$

$$\Theta = \tan^{-1}\frac{\xi\eta}{qR} \qquad Y_{11} = \frac{1}{R(R+\eta)} \qquad Y_{32} = \frac{2R+\eta}{R^3(R+\eta)^2} \qquad Z_{32} = \frac{\sin\delta}{R^3} - hY_{32}$$

Here,

$$\left(\frac{\eta}{R^3} + Y_{11}\right)\sin\delta - \frac{h}{R^3} = Y_{11}\sin\delta + \frac{\eta\sin\delta - h}{R^3} = Y_{11}\sin\delta - \frac{\tilde{c}}{R^3} \quad \rightarrow \quad u_2^C = (1-\alpha)\left(\frac{\cos\delta}{R} + 2qY_{11}\sin\delta\right) - \alpha\frac{\tilde{c}q}{R^3}$$

and

$$\left(\frac{1}{R} - \frac{q^2}{R^3}\right)\sin\delta - h\,(Y_{11} - q^2Y_{32}) = \frac{\xi^2 + \eta^2}{R^3}\sin\delta + h\left(Y_{11} - \xi^2 Y_{32} - \frac{\eta}{R^3}\right) = \xi^2\left(\frac{\sin\delta}{R^3} - hY_{32}\right) + \frac{\eta(\eta\sin\delta - h)}{R^3} + hY_{11}$$

$$= \xi^2 Z_{32} + \frac{\tilde{c}\eta}{R^3} + (q\cos\delta - z)Y_{11} \quad \rightarrow \quad u_3^C = (1-\alpha)qY_{11}\cos\delta - \alpha\left(\frac{\tilde{c}\eta}{R^3} - zY_{11} + \xi^2 Z_{32}\right)$$

The above three vectors correspond to the contents of the row of Strike-slip in Table 6.
(Evaluation of $\iint I_1^0 d\xi d\eta$ et al. will be done in the later section)

(2) Dip slip

Displacement due to a point dip-slip at $(0,0,-c)$ are given in Table 2 as follows.

$$u_A^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = & & \dfrac{\alpha}{2}\dfrac{3xpq}{R^5} \\ u_y = & \dfrac{1-\alpha}{2}\dfrac{s}{R^3} & +\dfrac{\alpha}{2}\dfrac{3ypq}{R^5} \\ u_z = & -\dfrac{1-\alpha}{2}\dfrac{t}{R^3} & +\dfrac{\alpha}{2}\dfrac{3dpq}{R^5} \end{pmatrix} \qquad u_B^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = -\dfrac{3xpq}{R^5} + \dfrac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta \\ u_y = -\dfrac{3ypq}{R^5} + \dfrac{1-\alpha}{\alpha}I_1^0\sin\delta\cos\delta \\ u_z = -\dfrac{3dpq}{R^5} + \dfrac{1-\alpha}{\alpha}I_5^0\sin\delta\cos\delta \end{pmatrix}$$

$$u_C^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = (1-\alpha)\dfrac{3xt}{R^5} & -\alpha\dfrac{15cxpq}{R^7} \\ u_y = -(1-\alpha)\dfrac{1}{R^3}\left(\cos 2\delta - \dfrac{3yt}{R^2}\right) & +\alpha\dfrac{3c}{R^5}\left(s - \dfrac{5ypq}{R^2}\right) \\ u_z = -(1-\alpha)\dfrac{A_3}{R^3}\sin\delta\cos\delta & +\alpha\dfrac{3c}{R^5}\left(t + \dfrac{5dpq}{R^2}\right) - \dfrac{3pq}{R^5} \end{pmatrix}$$

$$I_1^0 = y\left[\frac{1}{R(R+d)^2} - x^2\frac{3R+d}{R^3(R+d)^3}\right] \qquad I_2^0 = x\left[\frac{1}{R(R+d)^2} - y^2\frac{3R+d}{R^3(R+d)^3}\right]$$

$$I_3^0 = \frac{x}{R^3} - I_2^0 \qquad I_5^0 = \frac{1}{R(R+d)} - x^2\frac{2R+d}{R^3(R+d)^2}$$

where, $A_3 = 1 - \frac{3x^2}{R^2}$, $d = c - z$, $\begin{cases} p = y\cos\delta + d\sin\delta \\ q = y\sin\delta - d\cos\delta \end{cases}$, $pq = \frac{y^2 - d^2}{2}\sin 2\delta - yd\cos 2\delta$

$\begin{cases} s = p\sin\delta + q\cos\delta = y\sin 2\delta - d\cos 2\delta \\ t = p\cos\delta - q\sin\delta = y\cos 2\delta + d\sin 2\delta \end{cases}$, $R^2 = x^2 + y^2 + d^2 = x^2 + p^2 + q^2 = x^2 + s^2 + t^2$

Here, for the sake of simplicity, the term $-\frac{3cpq}{R^5}$ in the *z*-component of u_B^o was restored to $-\frac{3dpq}{R^5}$ and the term $-\frac{3pq}{R^5}$ was added to the *z*-component of u_C^o (see "Derivation of Table 2").

If we convert the displacement $(u_x,\ u_y,\ u_z)$ to $(u_1,\ u_2,\ u_3)$,

$$u_A^o=\frac{Mo}{2\pi\mu}\begin{pmatrix} u_1= & \frac{\alpha}{2}\frac{3xpq}{R^5} \\ u_2= & \frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3p^2q}{R^5} \\ u_3=- & \frac{1-\alpha}{2}\frac{p}{R^3}-\frac{\alpha}{2}\frac{3pq^2}{R^5}\end{pmatrix}\qquad u_B^o=\frac{Mo}{2\pi\mu}\begin{pmatrix} u_1=-\frac{3xpq}{R^5}+\frac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta \\ u_2=-\frac{3p^2q}{R^5}+\frac{1-\alpha}{\alpha}(I_1^0\cos\delta+I_5^0\sin\delta)\sin\delta\cos\delta \\ u_3=\ \frac{3pq^2}{R^5}-\frac{1-\alpha}{\alpha}(I_1^0\sin\delta-I_5^0\cos\delta)\sin\delta\cos\delta\end{pmatrix}$$

$$u_C^o=\frac{Mo}{2\pi\mu}\begin{pmatrix} u_1=\ (1-\alpha)\frac{3xt}{R^5} \qquad -\alpha\frac{15cxpq}{R^7} \\ u_2=-(1-\alpha)\frac{\cos\delta}{R^3}\left(\cos 2\delta-\frac{3yt}{R^2}-A_3\sin^2\delta\right)+\alpha\frac{3c}{R^5}\left(q-\frac{5p^2q}{R^2}\right)+\frac{3pq}{R^5}\sin\delta \\ u_3=\ (1-\alpha)\frac{\sin\delta}{R^3}\left(\cos 2\delta-\frac{3yt}{R^2}+A_3\cos^2\delta\right)-\alpha\frac{3c}{R^5}\left(p-\frac{5pq^2}{R^2}\right)+\frac{3pq}{R^5}\cos\delta\end{pmatrix}$$

Here, since $=p^2\cos^2\delta-q^2\sin^2\delta$,

$$\cos 2\delta-\frac{3yt}{R^2}-A_3\sin^2\delta=\cos 2\delta-\sin^2\delta-\frac{3(p^2\cos^2\delta-q^2\sin^2\delta-x^2\sin^2\delta)}{R^2}=\ \cos 2\delta+2\sin^2\delta-\frac{3p^2}{R^2}=1-\frac{3p^2}{R^2}$$

$$\cos 2\delta-\frac{3yt}{R^2}+A_3\cos^2\delta=\cos 2\delta+\cos^2\delta-\frac{3(p^2\cos^2\delta-q^2\sin^2\delta+x^2\cos^2\delta)}{R^2}=\ \cos 2\delta-2\cos^2\delta+\frac{3q^2}{R^2}=-1+\frac{3q^2}{R^2}$$

For the integration, we substitute
$$\begin{cases} x\ \rightarrow\ \xi \\ y\ \rightarrow\ \tilde{y}=\eta\cos\delta+q\sin\delta \\ d\ \rightarrow\ \tilde{d}=\eta\sin\delta-q\cos\delta \\ c\ \rightarrow\ \tilde{c}=\tilde{d}+z=\eta\sin\delta-h \\ p\ \rightarrow\ \eta \\ q\ \rightarrow\ q\end{cases}\qquad \begin{matrix} R^2=\xi^2+\eta^2+q^2=\xi^2+\tilde{y}^2+\tilde{d}^2 \\ h=q\cos\delta-z\end{matrix}$$

So, integrand becomes

$$u_A^o=\begin{pmatrix} u_1= & \frac{\alpha}{2}\frac{3\xi\eta q}{R^5} \\ u_2= & \frac{1-\alpha}{2}\frac{q}{R^3}+\frac{\alpha}{2}\frac{3\eta^2q}{R^5} \\ u_3=- & \frac{1-\alpha}{2}\frac{\eta}{R^3}-\frac{\alpha}{2}\frac{3\eta q^2}{R^5}\end{pmatrix}\quad u_B^o=\begin{pmatrix} u_1=-\frac{3\xi\eta q}{R^5}+\frac{1-\alpha}{\alpha}I_3^0\sin\delta\cos\delta \\ u_2=-\frac{3\eta^2q}{R^5}+\frac{1-\alpha}{\alpha}(I_1^0\cos\delta+I_5^0\sin\delta)\sin\delta\cos\delta \\ u_3=\ \frac{3\eta q^2}{R^5}-\frac{1-\alpha}{\alpha}(I_1^0\sin\delta-I_5^0\cos\delta)\sin\delta\cos\delta\end{pmatrix}$$

$$u_C^o=\begin{pmatrix} u_1=\ (1-\alpha)\frac{3\xi(\eta\cos\delta-q\sin\delta)}{R^5} \qquad -15\alpha\xi\eta q\frac{\eta\sin\delta-h}{R^7} \\ u_2=-(1-\alpha)\left(\frac{1}{R^3}-\frac{3\eta^2}{R^5}\right)\cos\delta+\alpha q(\eta\sin\delta-h)\left(\frac{3}{R^5}-\frac{15\eta^2}{R^7}\right)+\frac{3\eta q}{R^5}\sin\delta \\ u_3=-(1-\alpha)\left(\frac{1}{R^3}-\frac{3q^2}{R^5}\right)\sin\delta-\alpha\eta(\eta\sin\delta-h)\left(\frac{3}{R^5}-\frac{15q^2}{R^7}\right)+\frac{3\eta q}{R^5}\cos\delta\end{pmatrix}$$

$$I_1^0=\tilde{y}\left[\frac{1}{R(R+\tilde{d})^2}-\xi^2\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right]$$
$$I_2^0=\xi\left[\frac{1}{R(R+\tilde{d})^2}-\tilde{y}^2\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right]$$
$$I_3^0=\frac{\xi}{R^3}-I_2^0$$
$$I_5^0=\frac{1}{R(R+\tilde{d})}-\xi^2\frac{2R+\tilde{d}}{R^3(R+\tilde{d})^2}$$

At first, let us integrate with ξ (refer Appendix : Table of Integration)

$$\int u_A^o\,d\xi=\begin{pmatrix} u_1= & -\frac{\alpha}{2}\frac{\eta q}{R^3} \\ u_2=-\frac{1-\alpha}{2}qX_{11} & -\frac{\alpha}{2}\eta^2qX_{32} \\ u_3=\ \frac{1-\alpha}{2}\eta X_{11} & +\frac{\alpha}{2}\eta q^2X_{32}\end{pmatrix}\qquad \int u_B^o\,d\xi=\begin{pmatrix} u_1=\frac{\eta q}{R^3} \qquad +\frac{1-\alpha}{\alpha}\int I_3^0d\xi\ \sin\delta\cos\delta \\ u_2=\ \eta^2qX_{32}+\frac{1-\alpha}{\alpha}\int(I_1^0\cos\delta+I_5^0\sin\delta)d\xi\sin\delta\cos\delta \\ u_3=-\eta q^2X_{32}-\frac{1-\alpha}{\alpha}\int(I_1^0\sin\delta-I_5^0\cos\delta)d\xi\sin\delta\cos\delta\end{pmatrix}$$

$$\int u_C^o\,d\xi=\begin{pmatrix} u_1=-(1-\alpha)\frac{\eta\cos\delta-q\sin\delta}{R^3} \qquad +3\alpha\eta q\frac{\eta\sin\delta-h}{R^5} \\ u_2=\ (1-\alpha)(X_{11}-\eta^2X_{32})\cos\delta-\alpha q(\eta\sin\delta-h)(X_{32}-\eta^2X_{53})-\eta qX_{32}\sin\delta \\ u_3=\ (1-\alpha)(X_{11}-q^2X_{32})\sin\delta+\alpha\eta(\eta\sin\delta-h)(X_{32}-q^2X_{53})-\eta qX_{32}\cos\delta\end{pmatrix}$$

$$X_{11}=\frac{1}{R(R+\xi)}\qquad X_{32}=\frac{2R+\xi}{R^3(R+\xi)^2}\qquad X_{53}=\frac{8R^2+9R\xi+3\xi^2}{R^5(R+\xi)^3}$$

Next, let us integrate with η (refer Appendix : Table of Integration)

$$\iint u_A^o d\xi d\eta=\begin{pmatrix} u_1= & \frac{\alpha}{2}\frac{q}{R} \\ u_2=\frac{\Theta}{2} & +\frac{\alpha}{2}\eta qX_{11} \\ u_3=\frac{1-\alpha}{2}\ln(R+\xi) & -\frac{\alpha}{2}q^2X_{11}\end{pmatrix}\ \iint u_B^o d\xi d\eta=\begin{pmatrix} u_1=-\frac{q}{R} \qquad +\frac{1-\alpha}{\alpha}\iint I_3^0d\xi d\eta\ \sin\delta\cos\delta \\ u_2=-\eta qX_{11}-\Theta-\frac{1-\alpha}{\alpha}\iint(-I_1^0\cos\delta-I_5^0\sin\delta)d\xi d\eta\ \sin\delta\cos\delta \\ u_3=q^2X_{11} \qquad +\frac{1-\alpha}{\alpha}\iint(-I_1^0\sin\delta+I_5^0\cos\delta)d\xi d\eta\ \sin\delta\cos\delta\end{pmatrix}$$

$$\iint u_C^o d\xi d\eta = \begin{pmatrix} u_1 = (1-\alpha)\left(\frac{\cos\delta}{R} - qY_{11}\sin\delta\right) - \alpha q\left(\frac{\eta\sin\delta - h}{R^3} + Y_{11}\sin\delta\right) \\ u_2 = (1-\alpha)\eta X_{11}\cos\delta - \alpha q(X_{11}+\eta^2 X_{32})\sin\delta + \alpha q h\eta X_{32} + qX_{11}\sin\delta \\ u_3 = -(1-\alpha)(\eta X_{11}+\xi Y_{11})\sin\delta - \alpha(2\eta X_{11}+\xi Y_{11}-\eta q^2 X_{32})\sin\delta + \alpha h(X_{11}-q^2X_{32}) + qX_{11}\cos\delta \end{pmatrix} \quad \begin{matrix} \Theta = \tan^{-1}\frac{\xi\eta}{qR} \\ Y_{11} = \frac{1}{R(R+\eta)} \end{matrix}$$

Here,

$$u_1^C = (1-\alpha)\left(\frac{\cos\delta}{R} - qY_{11}\sin\delta\right) - \alpha q\left(\frac{\eta\sin\delta - h}{R^3} + Y_{11}\sin\delta\right) = (1-\alpha)\frac{\cos\delta}{R} - q\,Y_{11}\sin\delta - \alpha\frac{\tilde{c}q}{R^3}$$

$$u_2^C = (1-\alpha)(\eta\cos\delta + q\sin\delta)X_{11} - \alpha\eta q(\eta\sin\delta - h)X_{32} = (1-\alpha)\tilde{y}X_{11} - \alpha\tilde{c}\eta q X_{32}$$

$$u_3^C = -[\eta\sin\delta - q\cos\delta + \alpha(\eta\sin\delta - h)]X_{11} - \xi Y_{11}\sin\delta + \alpha q^2(\eta\sin\delta - h)X_{32} = -\tilde{d}X_{11} - \xi Y_{11}\sin\delta - \alpha\tilde{c}(X_{11} - q^2 X_{32})$$

The above three vectors correspond to the contents of the row of Dip-slip in Table 6.
(Evaluation of $\iint I_3^0 d\xi d\eta$ et al. will be done in the later section)

(3) Tensile

Displacement due to a point tensile fault at $(0,0,-c)$ are given in Table 2 as follows.

$$u_A^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = \frac{1-\alpha}{2}\frac{x}{R^3} - \frac{\alpha}{2}\frac{3xq^2}{R^5} \\ u_y = \frac{1-\alpha}{2}\frac{t}{R^3} - \frac{\alpha}{2}\frac{3yq^2}{R^5} \\ u_z = \frac{1-\alpha}{2}\frac{s}{R^3} - \frac{\alpha}{2}\frac{3dq^2}{R^5} \end{pmatrix} \qquad u_B^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = \frac{3xq^2}{R^5} - \frac{1-\alpha}{\alpha}I_3^0\sin^2\delta \\ u_y = \frac{3yq^2}{R^5} - \frac{1-\alpha}{\alpha}I_1^0\sin^2\delta \\ u_z = \frac{3dq^2}{R^5} - \frac{1-\alpha}{\alpha}I_5^0\sin^2\delta \end{pmatrix}$$

$$u_C^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_x = -(1-\alpha)\frac{3xs}{R^5} + \alpha\frac{15cxq^2}{R^7} - \alpha\frac{3xz}{R^5} \\ u_y = (1-\alpha)\frac{1}{R^3}\left(\sin2\delta - \frac{3ys}{R^2}\right) + \alpha\frac{3c}{R^5}\left(t - y + \frac{5yq^2}{R^2}\right) - \alpha\frac{3yz}{R^5} \\ u_z = -(1-\alpha)\frac{1}{R^3}(1 - A_3\sin^2\delta) - \alpha\frac{3c}{R^5}\left(s - d + \frac{5dq^2}{R^2}\right) + \alpha\frac{3dz}{R^5} + \frac{3q^2}{R^5} \end{pmatrix}$$

$$I_1^0 = y\left[\frac{1}{R(R+d)^2} - x^2\frac{3R+d}{R^3(R+d)^3}\right]$$
$$I_2^0 = x\left[\frac{1}{R(R+d)^2} - y^2\frac{3R+d}{R^3(R+d)^3}\right]$$
$$I_3^0 = \frac{x}{R^3} - I_2^0$$
$$I_5^0 = \frac{1}{R(R+d)} - x^2\frac{2R+d}{R^3(R+d)^2}$$

where, $A_3 = 1 - \frac{3x^2}{R^2}$, $d = c - z$, $\begin{cases} p = y\cos\delta + d\sin\delta \\ q = y\sin\delta - d\cos\delta \end{cases}$, $pq = \frac{y^2 - d^2}{2}\sin2\delta - yd\cos2\delta$

$\begin{cases} s = p\sin\delta + q\cos\delta = y\sin2\delta - d\cos2\delta \\ t = p\cos\delta - q\sin\delta = y\cos2\delta + d\sin2\delta \end{cases}$, $R^2 = x^2 + y^2 + d^2 = x^2 + p^2 + q^2 = x^2 + s^2 + t^2$

Here, for the sake of simplicity, the term $\frac{3cq^2}{R^5}$ in the z-component of u_B^o was restored to $\frac{3dq^2}{R^5}$ and the term $\frac{3q^2}{R^5}$ was added to the z-component of u_C^o (see "Derivation of Table 2").

If we convert the displacement (u_x, u_y, u_z) to (u_1, u_2, u_3) ,

$$u_A^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_1 = \frac{1-\alpha}{2}\frac{x}{R^3} - \frac{\alpha}{2}\frac{3xq^2}{R^5} \\ u_2 = \frac{1-\alpha}{2}\frac{p}{R^3} - \frac{\alpha}{2}\frac{3pq^2}{R^5} \\ u_3 = \frac{1-\alpha}{2}\frac{q}{R^3} + \frac{\alpha}{2}\frac{3q^3}{R^5} \end{pmatrix} \qquad u_B^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_1 = \frac{3xq^2}{R^5} - \frac{1-\alpha}{\alpha}I_3^0\sin^2\delta \\ u_2 = \frac{3pq^2}{R^5} - \frac{1-\alpha}{\alpha}(I_1^0\cos\delta + I_5^0\sin\delta)\sin^2\delta \\ u_3 = -\frac{3q^3}{R^5} + \frac{1-\alpha}{\alpha}(I_1^0\sin\delta - I_5^0\cos\delta)\sin^2\delta \end{pmatrix}$$

$$u_C^o = \frac{Mo}{2\pi\mu}\begin{pmatrix} u_1 = -(1-\alpha)\frac{3xs}{R^5} + \alpha\frac{15cxq^2}{R^7} - \alpha\frac{3xz}{R^5} \\ u_2 = (1-\alpha)\frac{1}{R^3}\left[\left(\sin2\delta - \frac{3ys}{R^2}\right)\cos\delta + (1 - A_3\sin^2\delta)\sin\delta\right] + \alpha\frac{15cpq^2}{R^7} - \alpha\frac{3pz}{R^5} - \frac{3q^2}{R^5}\sin\delta \\ u_3 = -(1-\alpha)\frac{1}{R^3}\left[\left(\sin2\delta - \frac{3ys}{R^2}\right)\sin\delta - (1 - A_3\sin^2\delta)\cos\delta\right] + \alpha\frac{3cq}{R^5}\left(2 - \frac{5q^2}{R^2}\right) + \alpha\frac{3qz}{R^5} - \frac{3q^2}{R^5}\cos\delta \end{pmatrix}$$

Here, since $= (p^2 + q^2)\sin\delta\cos\delta + pq$,

$$\left(\sin2\delta - \frac{3ys}{R^2}\right)\cos\delta + (1 - A_3\sin^2\delta)\sin\delta = 2\sin\delta\cos^2\delta - \frac{3(p^2+q^2)\sin\delta\cos^2\delta - 3x^2\sin^3\delta}{R^2} - \frac{3pq}{R^2}\cos\delta = \frac{3x^2}{R^2}\sin\delta - \frac{3pq}{R^2}\cos\delta$$

$$\left(\sin2\delta - \frac{3ys}{R^2}\right)\sin\delta - (1 - A_3\sin^2\delta)\cos\delta = 2\sin^2\delta\cos\delta - \cos^3\delta - \frac{3(p^2+q^2) + 3x^2}{R^2}\sin^2\delta\cos\delta - \frac{3pq}{R^2}\sin\delta = -\cos\delta - \frac{3pq}{R^2}\sin\delta$$

For the integration, we substitute
$$\left\{\begin{array}{l} x \to \xi \\ y \to \tilde{y} = \eta\cos\delta + q\sin\delta \\ d \to \tilde{d} = \eta\sin\delta - q\cos\delta \\ c \to \tilde{c} = \tilde{d} + z = \eta\sin\delta - h \\ p \to \eta \\ q \to q \end{array}\right. \qquad \begin{array}{l} R^2 = \xi^2+\eta^2+q^2 = \xi^2+\tilde{y}^2+\tilde{d}^2 \\ \\ h = q\cos\delta - z \end{array}$$

So, integrand becomes

$$u_A^o = \left(\begin{array}{l} u_1 = \dfrac{1-\alpha}{2}\dfrac{\xi}{R^3} - \dfrac{\alpha}{2}\dfrac{3\xi q^2}{R^5} \\ u_2 = \dfrac{1-\alpha}{2}\dfrac{\eta}{R^3} - \dfrac{\alpha}{2}\dfrac{3\eta q^2}{R^5} \\ u_3 = \dfrac{1-\alpha}{2}\dfrac{q}{R^3} + \dfrac{\alpha}{2}\dfrac{3q^3}{R^5} \end{array}\right) \qquad u_B^o = \left(\begin{array}{l} u_1 = \dfrac{3\xi q^2}{R^5} - \dfrac{1-\alpha}{\alpha} I_3^0 \sin^2\delta \\ u_2 = \dfrac{3\eta q^2}{R^5} - \dfrac{1-\alpha}{\alpha}(I_1^0\cos\delta + I_5^0\sin\delta)\sin^2\delta \\ u_3 = -\dfrac{3q^3}{R^5} + \dfrac{1-\alpha}{\alpha}(I_1^0\sin\delta - I_5^0\cos\delta)\sin^2\delta \end{array}\right)$$

$$\begin{array}{l} I_1^0 = \tilde{y}\left[\dfrac{1}{R(R+\tilde{d})^2} - \xi^2\dfrac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right] \\ I_2^0 = \xi\left[\dfrac{1}{R(R+\tilde{d})^2} - \tilde{y}^2\dfrac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right] \\ I_3^0 = \dfrac{\xi}{R^3} - I_2^0 \\ I_5^0 = \dfrac{1}{R(R+\tilde{d})} - \xi^2\dfrac{2R+\tilde{d}}{R^3(R+\tilde{d})^2} \end{array}$$

$$u_C^o = \left(\begin{array}{l} u_1 = -(1-\alpha)\dfrac{3\xi(\eta\sin\delta + q\cos\delta)}{R^5} + 15\alpha\dfrac{(\eta\sin\delta - h)\xi q^2}{R^7} - \alpha\dfrac{3\xi z}{R^5} \\ u_2 = -(1-\alpha)\left(-\dfrac{3\xi^2}{R^5}\sin\delta + \dfrac{3\eta q}{R^5}\cos\delta\right) + 15\alpha\dfrac{(\eta\sin\delta - h)\eta q^2}{R^7} - \alpha\dfrac{3\eta z}{R^5} - \dfrac{3q^2}{R^5}\sin\delta \\ u_3 = (1-\alpha)\left(\dfrac{\cos\delta}{R^3} + \dfrac{3\eta q}{R^5}\sin\delta\right) + \alpha q(\eta\sin\delta - h)\left(\dfrac{6}{R^5} - \dfrac{15q^2}{R^7}\right) + \alpha\dfrac{3qz}{R^5} - \dfrac{3q^2}{R^5}\cos\delta \end{array}\right)$$

At first, let us integrate with ξ (refer Appendix : Table of Integration)

$$\int u_A^o\,d\xi = \left(\begin{array}{l} u_1 = -\dfrac{1-\alpha}{2}\dfrac{1}{R} + \dfrac{\alpha}{2}\dfrac{q^2}{R^3} \\ u_2 = -\dfrac{1-\alpha}{2}\eta X_{11} + \dfrac{\alpha}{2}\eta q^2 X_{32} \\ u_3 = -\dfrac{1-\alpha}{2} q X_{11} - \dfrac{\alpha}{2} q^3 X_{32} \end{array}\right) \qquad \int u_B^o\,d\xi = \left(\begin{array}{l} u_1 = -\dfrac{q^2}{R^3} - \dfrac{1-\alpha}{\alpha}\displaystyle\int I_3^0 d\xi \sin^2\delta \\ u_2 = -\eta q^2 X_{32} - \dfrac{1-\alpha}{\alpha}\displaystyle\int (I_1^0\cos\delta + I_5^0\sin\delta)d\xi \sin^2\delta \\ u_3 = q^3 X_{32} + \dfrac{1-\alpha}{\alpha}\displaystyle\int (I_1^0\sin\delta - I_5^0\cos\delta)d\xi \sin^2\delta \end{array}\right)$$

$$\int u_C^o\,d\xi = \left(\begin{array}{l} u_1 = (1-\alpha)\dfrac{\eta\sin\delta + q\cos\delta}{R^3} - 3\alpha q^2\dfrac{\eta\sin\delta - h}{R^5} + \alpha\dfrac{z}{R^3} \\ u_2 = -(1-\alpha)\left(\dfrac{\xi}{R^3}\sin\delta + X_{11}\sin\delta - \eta q X_{32}\cos\delta\right) - \alpha\eta q^2(\eta\sin\delta - h)X_{53} + \alpha\eta z X_{32} + q^2 X_{32}\sin\delta \\ u_3 = -(1-\alpha)(X_{11}\cos\delta + \eta q X_{32}\sin\delta) - \alpha q(\eta\sin\delta - h)(2X_{32} - q^2 X_{53}) - \alpha q z X_{32} + q^2 X_{32}\cos\delta \end{array}\right)$$

$$X_{11} = \frac{1}{R(R+\xi)} \qquad X_{32} = \frac{2R+\xi}{R^3(R+\xi)^2} \qquad X_{53} = \frac{8R^2+9R\xi+3\xi^2}{R^5(R+\xi)^3}$$

Next, let us integrate with η (refer Appendix : Table of Integration)

$$\iint u_A^o\,d\xi d\eta = \left(\begin{array}{l} u_1 = -\dfrac{1-\alpha}{2}\ln(R+\eta) - \dfrac{\alpha}{2}q^2 Y_{11} \\ u_2 = -\dfrac{1-\alpha}{2}\ln(R+\xi) - \dfrac{\alpha}{2}q^2 X_{11} \\ u_3 = \dfrac{\Theta}{2} - \dfrac{\alpha}{2}q(\eta X_{11} + \xi Y_{11}) \end{array}\right)$$

$$\iint u_B^o\,d\xi d\eta = \left(\begin{array}{l} u_1 = q^2 Y_{11} - \dfrac{1-\alpha}{\alpha}\displaystyle\iint I_3^0 d\xi d\eta \sin^2\delta \\ u_2 = q^2 X_{11} + \dfrac{1-\alpha}{\alpha}\displaystyle\iint (-I_1^0\cos\delta - I_5^0\sin\delta)d\xi d\eta \sin^2\delta \\ u_3 = q(\eta X_{11} + \xi Y_{11}) - \Theta - \dfrac{1-\alpha}{\alpha}\displaystyle\iint (-I_1^0\sin\delta + I_5^0\cos\delta)d\xi d\eta \sin^2\delta \end{array}\right)$$

$$\Theta = \tan^{-1}\frac{\xi\eta}{qR} \qquad Y_{11} = \frac{1}{R(R+\eta)} \qquad Y_{32} = \frac{2R+\eta}{R^3(R+\eta)^2} \qquad Z_{32} = \frac{\sin\delta}{R^3} - hY_{32}$$

$$\iint u_C^o\,d\xi d\eta = \left(\begin{array}{l} u_1 = -(1-\alpha)\left(\dfrac{\sin\delta}{R} + qY_{11}\cos\delta\right) + \alpha q^2\left(\dfrac{\sin\delta}{R^3} - hY_{32}\right) - \alpha z Y_{11} \\ u_2 = (1-\alpha)\left(\xi Y_{11}\sin\delta + \dfrac{\Theta}{q}\sin\delta - qX_{11}\cos\delta\right) + (1-\alpha)\left(\eta X_{11} + \xi Y_{11} - \dfrac{\Theta}{q}\right)\sin\delta + \alpha q^2(\eta\sin\delta - h)X_{32} - \alpha z X_{11} \\ u_3 = -(1-\alpha)\left(\dfrac{\Theta}{q}\cos\delta + qX_{11}\sin\delta\right) + \alpha q(2X_{11} - q^2 X_{32})\sin\delta + (1-\alpha)\left(\eta X_{11} + \xi Y_{11} - \dfrac{\Theta}{q}\right)\cos\delta - \alpha q h(\eta X_{32} + \xi Y_{32}) \end{array}\right)$$

Here,

$$\begin{aligned} u_2^C &= (1-\alpha)[2\xi Y_{11}\sin\delta + (\eta\cos\delta + q\sin\delta)X_{11}] - \alpha[zX_{11} - q^2(\eta\sin\delta - h)X_{32}] \\ &= (1-\alpha)2\xi Y_{11}\sin\delta + \tilde{d}X_{11} - \alpha(\tilde{d}+z)X_{11} - \alpha\tilde{c}q^2X_{32} = (1-\alpha)2\xi Y_{11}\sin\delta + \tilde{d}X_{11} - \alpha\tilde{c}(X_{11} - q^2X_{32}) \end{aligned}$$

$$\begin{aligned} u_3^C &= (1-\alpha)(qX_{11}\sin\delta + \eta X_{11}\cos\delta + \xi Y_{11}\cos\delta) - \alpha q[(q^2X_{32} - 2X_{11})\sin\delta + \eta h X_{32} + \xi h Y_{32}] \\ &= (1-\alpha)(\tilde{y}X_{11} + \xi Y_{11}\cos\delta) + \alpha q\left[(\eta^2 X_{32} + \frac{\xi}{R^3})\sin\delta - \eta h X_{32} - \xi h Y_{32}\right] \\ &= (1-\alpha)(\tilde{y}X_{11} + \xi Y_{11}\cos\delta) + \alpha q\left[\eta(\eta\sin\delta - h)X_{32} + \xi\left(\frac{\sin\delta}{R^3} - hY_{32}\right)\right] \end{aligned}$$

The above three vectors correspond to the contents of the row of Tensile in Table 6.

(4) Evaluation of $\iint I_1^0 d\xi d\eta \sim \iint I_5^0 d\xi d\eta$

For the integration, we substitute $\begin{cases} x \to \xi \\ y \to \tilde{y} = \eta\cos\delta + q\sin\delta \\ d \to \tilde{d} = \eta\sin\delta - q\cos\delta \end{cases}$ to I_1^0 through I_5^0 of the point solution in Table 2.

So, the integrands and their integral with ξ become as follows (refer Appendix : Table of Integration)

$$\begin{cases} I_1^0 = \tilde{y}\left[\dfrac{1}{R(R+\tilde{d})^2} - \xi^2\dfrac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right] \\ I_2^0 = \xi\left[\dfrac{1}{R(R+\tilde{d})^2} - \tilde{y}^2\dfrac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}\right] \\ I_3^0 = \dfrac{\xi}{R^3} - I_2^0 \\ I_4^0 = -\xi\tilde{y}\dfrac{2R+\tilde{d}}{R^3(R+\tilde{d})^2} \\ I_5^0 = \dfrac{1}{R(R+\tilde{d})} - \xi^2\dfrac{2R+\tilde{d}}{R^3(R+\tilde{d})^2} \end{cases} \qquad \begin{cases} \displaystyle\int I_1^0\,d\xi = \frac{\xi\tilde{y}}{R(R+\tilde{d})^2} \\ \displaystyle\int I_2^0\,d\xi = -\frac{1}{R+\tilde{d}} + \frac{\tilde{y}^2}{R(R+\tilde{d})^2} \\ \displaystyle\int I_3^0\,d\xi = -\frac{1}{R} - \int I_2^0\,d\xi \\ \displaystyle\int I_4^0\,d\xi = \frac{\tilde{y}}{R(R+\tilde{d})} \\ \displaystyle\int I_5^0\,d\xi = \frac{\xi}{R(R+\tilde{d})} \end{cases}$$

Next, let us integrate with η (refer Appendix : Table of Integration)

(a) $I_5 \equiv \displaystyle\iint I_5^0 d\xi d\eta = \int \frac{\xi}{R(R+\tilde{d})}d\eta$ $\qquad$ ($R^2 = \eta^2 + X^2$, $X^2 = \xi^2 + q^2$, $\tilde{d} = \eta\sin\delta - q\cos\delta$)

< **Case 1** > $\boldsymbol{X \neq 0}$

By changing integral variable $\eta \to t = \dfrac{R-X}{\eta} = \dfrac{\eta}{R+X}$ $(X^2 = \xi^2 + q^2)$, $\quad R = \dfrac{1+t^2}{1-t^2}X,\ \ \eta = \dfrac{2t}{1-t^2}X,\ \ d\eta = \dfrac{2(1+t^2)}{(1-t^2)^2}Xdt$

and from the formula $\displaystyle\int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{1}{\sqrt{4ac-b^2}}\tan^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } b^2 < 4ac \\ -\dfrac{2}{2ax+b} & \text{for } b^2 = 4ac \end{cases}$ $\qquad b^2 - 4ac = -4\xi^2\cos^2\delta$ ⇧

$$\int \frac{\xi}{R(R+\tilde{d})}d\eta = \int \frac{\xi}{\frac{1+t^2}{1-t^2}X\left(\frac{1+t^2}{1-t^2}X + \frac{2t\sin\delta}{1-t^2}X - q\cos\delta\right)}\frac{2(1+t^2)}{(1-t^2)^2}Xdt = \int \frac{2\xi}{(X+q\cos\delta)t^2 + (2X\sin\delta)t + (X-q\cos\delta)}dt$$

$$= \begin{cases} \dfrac{2\xi}{|\xi\cos\delta|}\tan^{-1}\dfrac{(X+q\cos\delta)t + X\sin\delta}{|\xi\cos\delta|} = \dfrac{2}{\cos\delta}\tan^{-1}\dfrac{\eta(X+q\cos\delta) + X(R+X)\sin\delta}{\xi(R+X)\cos\delta} & \text{for } \xi\cos\delta \neq 0 \quad (1) \\ -\dfrac{2\xi}{(X+q\cos\delta)t + X\sin\delta} = -\dfrac{2\xi(R+X)}{\eta(X+q\cos\delta) + X(R+X)\sin\delta} & \text{for } \xi\cos\delta = 0 \end{cases}$$

In the latter case, $I_5 = 0$ if $\xi = 0$

while if $\cos\delta = 0$, $\sin\delta = \pm 1$ and $I_5 = \displaystyle\int \frac{\xi}{R(R\pm\eta)}d\eta = \mp\frac{\xi}{R\pm\eta} = -\frac{\xi\sin\delta}{R+\eta\sin\delta} = -\frac{\xi\sin\delta}{R+\tilde{d}}$ $\qquad$ (2)

< **Case 2** > $\boldsymbol{X = 0}$ $\qquad (\xi = q = 0,\ R = |\eta|)$

$$I_5 = \int \frac{\xi}{R(R+\tilde{d})}d\eta = \int \frac{\xi}{|\eta|(|\eta| + \eta\sin\delta)}d\eta = 0$$

So, as a whole, $I_5 = 0$ when $\xi = 0$. Otherwise I_5 takes either of (1) or (2) depending on $\cos\delta = 0$ or not.

(b) $I_4 \equiv \displaystyle\iint I_4^0 d\xi d\eta = \int \frac{\tilde{y}}{R(R+\tilde{d})}d\eta$ $\qquad$ ($R^2 = \xi^2 + \tilde{y}^2 + \tilde{d}^2$, $\tilde{y} = \eta\cos\delta + q\sin\delta$, $\tilde{d} = \eta\sin\delta - q\cos\delta$)

< **Case 1** > $\boldsymbol{\cos\delta \neq 0}$

Since $\tilde{y} = \dfrac{1}{\cos\delta}(\eta - \tilde{d}\sin\delta)$

$$I_4 = \frac{1}{\cos\delta}\int \frac{\eta - \tilde{d}\sin\delta}{R(R+\tilde{d})}d\eta = \frac{1}{\cos\delta}\int\left(\frac{\eta}{R(R+\tilde{d})} + \frac{\sin\delta}{R+\tilde{d}} - \frac{\sin\delta}{R}\right)d\eta = \frac{1}{\cos\delta}\left[\ln(R+\tilde{d}) - \sin\delta\ln(R+\eta)\right]$$

< **Case 2** > $\boldsymbol{\cos\delta = 0}$ $\qquad (\sin\delta = \pm 1)$

Since $\tilde{y} = \pm q$ and $\tilde{d} = \pm\eta$, $\qquad I_4 = \displaystyle\int \frac{\pm q}{R(R\pm\eta)}d\eta = -\frac{q}{R\pm\eta} = -\frac{q}{R+\tilde{d}}$

$(c)\ \ I_1 \equiv \iint I_1^0 d\xi d\eta = \int \frac{\xi\tilde{y}}{R(R+\tilde{d})^2} d\eta \qquad (R^2 = \xi^2 + \tilde{y}^2 + \tilde{d}^2,\ \tilde{y} = \eta\cos\delta + q\sin\delta,\ \ \tilde{d} = \eta\sin\delta - q\cos\delta)$

< **Case 1** > $\cos\delta \neq 0$

Since $\tilde{y} = \frac{1}{\cos\delta}(\eta - \tilde{d}\sin\delta)$

$$I_1 = \frac{\xi}{\cos\delta}\int \frac{\eta - \tilde{d}\sin\delta}{R(R+\tilde{d})^2} d\eta = \frac{\xi}{\cos\delta}\int\left(\frac{\eta}{R(R+\tilde{d})^2} + \frac{\sin\delta}{(R+\tilde{d})^2} - \frac{\sin\delta}{R(R+\tilde{d})}\right) d\eta = -\frac{1}{\cos\delta}\left(\frac{\xi}{R+\tilde{d}} + I_5\sin\delta\right)$$

< **Case 2** > $\cos\delta = 0 \qquad (\sin\delta = \pm 1)$

Since $\tilde{y} = \pm q$ and $\tilde{d} = \pm\eta$, $\qquad I_1 = \int \frac{\pm\xi q}{R(R\pm\eta)^2} d\eta = -\frac{\xi q}{2(R\pm\eta)^2} = -\frac{\xi q}{2(R+\tilde{d})^2}$

$$(d)\ \ I_2 \equiv \iint I_2^0 d\xi d\eta = -\int\left(\frac{1}{R+\tilde{d}} - \frac{\tilde{y}^2}{R(R+\tilde{d})^2}\right) d\eta$$

< **Case 1** > $\cos\delta \neq 0$

Since $\tilde{y} = \frac{1}{\cos\delta}(\eta - \tilde{d}\sin\delta)$

$$I_2 = -\int\left[\frac{1}{R+\tilde{d}} - \frac{1}{\cos\delta}\frac{\eta\tilde{y}}{R(R+\tilde{d})^2} + \frac{\sin\delta}{\cos\delta}\left(\frac{\tilde{y}}{R(R+\tilde{d})} - \frac{\tilde{y}}{(R+\tilde{d})^2}\right)\right] d\eta$$
$$= -\frac{1}{\cos\delta}\int\left[\frac{\cos\delta}{R+\tilde{d}} - \frac{\eta\tilde{y}}{R(R+\tilde{d})^2} - \frac{\tilde{y}\sin\delta}{(R+\tilde{d})^2}\right] - \frac{\sin\delta}{\cos\delta}I_4 = -\frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} - \frac{\sin\delta}{\cos\delta}I_4$$

< **Case 2** > $\cos\delta = 0 \qquad (\sin\delta = \pm 1)$

Since $\tilde{y} = \pm q$, $\tilde{d} = \pm\eta$ and $-\ln(R-\eta) = \ln(R+\eta) - \ln(R^2-\eta^2)$

$$I_2 = -\int\left(\frac{1}{R\pm\eta} - \frac{q^2}{R(R\pm\eta)^2}\right) d\eta = -\frac{1}{2}\left(\frac{\eta}{R\pm\eta} \pm \ln(R\pm\eta)\right) \mp \frac{q^2}{2(R\pm\eta)^2} = -\frac{1}{2}\left(\frac{\eta}{R+\tilde{d}} + \ln(R+\eta)\right) - \frac{\tilde{y}q}{2(R+\tilde{d})^2}$$

$$(e)\ \ I_3 \equiv \iint I_3^0 d\xi d\eta = -\int\frac{1}{R}d\eta - \iint I_2^0 d\xi d\eta \ = -\ln(R+\eta) - I_2$$

As a conclusion, the latter part of $\boldsymbol{u_B}$ including $\boldsymbol{I_1}$ through $\boldsymbol{I_4}$ in Table 6 are given as follows ($\cos\delta \neq 0$).

(1) Strike-slip

$$u_1{}^B:\ \ \boldsymbol{I_1} \equiv \iint I_1^0 d\xi d\eta = -\frac{1}{\cos\delta}\left(\frac{\xi}{R+\tilde{d}} + I_5\sin\delta\right) = -\frac{\xi}{R+\tilde{d}}\cos\delta - \boldsymbol{I_4}\sin\delta \qquad \left(\text{since}\ \ I_5 = -\frac{\xi}{R+\tilde{d}}\sin\delta - \boldsymbol{I_4}\cos\delta\right)$$

$$u_2{}^B:\ \ \iint(-I_2^0\cos\delta - I_4^0\sin\delta)\, d\xi d\eta = \left(\frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} + \frac{\sin\delta}{\cos\delta}I_4\right)\cos\delta - I_4\sin\delta = \frac{\tilde{y}}{R+\tilde{d}}$$

$$u_3{}^B:\ \ \boldsymbol{I_2} \equiv \iint(-I_2^0\cos\delta + I_4^0\sin(\delta))d\xi d\eta = \left(\frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} + \frac{\sin\delta}{\cos\delta}I_4\right)\sin\delta + I_4\cos\delta = \frac{\sin\delta}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} + \frac{1}{\cos\delta}I_4$$
$$= \frac{\sin\delta}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} + \frac{1}{\cos^2\delta}\left[\ln(R+\tilde{d}) - \sin\delta\ln(R+\eta)\right] = \ln(R+\tilde{d}) + \boldsymbol{I_3}\sin\delta$$

(2) Dip-slip and Tensile

$$u_1{}^B:\ \ \boldsymbol{I_3} \equiv \iint I_3^0 d\xi d\eta = -\ln(R+\eta) - I_2 = -\ln(R+\eta) + \frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} + \frac{\sin\delta}{\cos\delta}I_4 = \frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} - \frac{1}{\cos^2\delta}\left[\ln(R+\eta) - \sin\delta\ln(R+\tilde{d})\right]$$

$$u_2{}^B:\ \ \iint(-I_1^0\cos\delta - I_5^0\sin\delta)\, d\xi d\eta = \left(\frac{1}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{\sin\delta}{\cos\delta}I_5\right)\cos\delta - I_5\sin\delta = \frac{\xi}{R+\tilde{d}}$$

$$u_3{}^B:\ \ \boldsymbol{I_4} \equiv \iint(-I_1^0\sin\delta + I_5^0\cos(\delta))d\xi d\eta = \left(\frac{1}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{\sin\delta}{\cos\delta}I_5\right)\sin\delta + I_5\cos\delta = \frac{\sin\delta}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{1}{\cos\delta}I_5$$
$$= \frac{\sin\delta}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{2}{\cos^2\delta}\tan^{-1}\frac{\eta(X+q\cos\delta) + X(R+X)\sin\delta}{\xi(R+X)\cos\delta}$$

In case of $\cos\delta = 0$, $\boldsymbol{I_3}$ and $\boldsymbol{I_4}$ should be given as follows.

$$\boldsymbol{I_3} = -\ln(R+\eta) - I_2 = -\ln(R+\eta) + \frac{1}{2}\left(\frac{\eta}{R+\tilde{d}} + \ln(R+\eta)\right) + \frac{\tilde{y}q}{2(R+\tilde{d})^2} = \frac{1}{2}\left(\frac{\eta}{R+\tilde{d}} + \frac{\tilde{y}q}{(R+\tilde{d})^2} - \ln(R+\eta)\right)$$

$$\boldsymbol{I_4} = -I_1\sin\delta = \frac{\xi q}{2(R+\tilde{d})^2}\sin\delta = \frac{\xi\tilde{y}}{2(R+\tilde{d})^2}$$

Appendix : Table of Integration

$$R=\sqrt{\xi^2+\eta^2+q^2}=\sqrt{\xi^2+\tilde{y}^2+\tilde{d}^2} \qquad \begin{cases}\tilde{y}=\eta\cos\delta+q\sin\delta\\ \tilde{d}=\eta\sin\delta-q\cos\delta\end{cases}$$

f	$\int f\,d\xi$	$\int f\,d\eta$
$1/R$	$\ln(R+\xi)$	$\ln(R+\eta)$
$1/R^3$	$-X_{11}$	$-Y_{11}$
$3/R^5$	$-X_{32}$	$-Y_{32}$
$15/R^7$	$-X_{53}$	$-Y_{53}$
ξ/R^3	$-1/R$	$-\xi Y_{11}$
$3\xi/R^5$	$-1/R^3$	$-\xi Y_{32}$
$15\xi/R^7$	$-3/R^5$	$-\xi Y_{53}$
η/R^3	$-\eta X_{11}$	$-1/R$
$3\eta/R^5$	$-\eta X_{32}$	$-1/R^3$
$15\eta/R^7$	$-\eta X_{53}$	$-3/R^5$
$3\xi^2/R^5$	$-\dfrac{\xi}{R^3}-X_{11}$	$-\xi^2 Y_{32}$
$15\xi^2/R^7$	$-\dfrac{3\xi}{R^5}-X_{32}$	$-\xi^2 Y_{53}$
$3\eta^2/R^5$	$-\eta^2 X_{32}$	$-\dfrac{\eta}{R^3}-Y_{11}$
$15\eta^2/R^7$	$-\eta^2 X_{53}$	$-\dfrac{3\eta}{R^5}-Y_{32}$
X_{11}	$-\dfrac{1}{R+\xi}$	$-\dfrac{1}{q}\tan^{-1}\dfrac{\xi\eta}{qR}$ (*)
X_{32}	$-\dfrac{1}{R(R+\xi)^2}$	$\dfrac{1}{q^2}(\eta X_{11}+\xi Y_{11})-\dfrac{1}{q^3}\tan^{-1}\dfrac{\xi\eta}{qR}$
X_{53}	$-\dfrac{3R+\xi}{R^3(R+\xi)^3}$	$\dfrac{1}{q^2}(\eta X_{32}+\xi Y_{32})+\dfrac{3}{q^4}(\eta X_{11}+\xi Y_{11})-\dfrac{3}{q^5}\tan^{-1}\dfrac{\xi\eta}{qR}$
ξX_{11}	$\dfrac{1}{2}\left(\dfrac{R}{R+\xi}+\ln(R+\xi)\right)$	$-\dfrac{\xi}{q}\tan^{-1}\dfrac{\xi\eta}{qR}$
ξX_{32}	$\dfrac{1}{2(R+\xi)^2}-X_{11}$	$\dfrac{\xi}{q^2}(\eta X_{11}+\xi Y_{11})-\dfrac{\xi}{q^3}\tan^{-1}\dfrac{\xi\eta}{qR}$
ξX_{53}	$\dfrac{1}{R(R+\xi)^3}-X_{32}$	$\dfrac{\xi}{q^2}(\eta X_{32}+\xi Y_{32})+\dfrac{3\xi}{q^4}(\eta X_{11}+\xi Y_{11})-\dfrac{3\xi}{q^5}\tan^{-1}\dfrac{\xi\eta}{qR}$
ηX_{11}	$-\dfrac{\eta}{R+\xi}$	$\ln(R+\xi)$
ηX_{32}	$-\dfrac{\eta}{R(R+\xi)^2}$	$-X_{11}$
ηX_{53}	$-\dfrac{\eta(3R+\xi)}{R^3(R+\xi)^3}$	$-X_{32}$
$\eta^2 X_{32}$	$-\dfrac{\eta^2}{R(R+\xi)^2}$	$-\eta X_{11}-\dfrac{1}{q}\tan^{-1}\dfrac{\xi\eta}{qR}$
$\eta^2 X_{53}$	$-\dfrac{\eta^2(3R+\xi)}{R^3(R+\xi)^3}$	$-\eta X_{32}+\dfrac{1}{q^2}(\eta X_{11}+\xi Y_{11})-\dfrac{1}{q^3}\tan^{-1}\dfrac{\xi\eta}{qR}$
$\eta^3 X_{32}$	$-\dfrac{\eta^3}{R(R+\xi)^2}$	$2\ln(R+\xi)-\eta^2 X_{11}$
$\eta^3 X_{53}$	$-\dfrac{\eta^3(3R+\xi)}{R^3(R+\xi)^3}$	$-2X_{11}-\eta^2 X_{32}$

$\frac{1}{R+\eta}$	$\ln(R+\xi)+\frac{\eta}{q}\left(\tan^{-1}\frac{\xi\eta}{qR}-\tan^{-1}\frac{\xi}{q}\right)$	$\frac{1}{2}\left(\frac{\eta}{R+\eta}+\ln(R+\eta)\right)$
$\frac{1}{R-\eta}$	$-\ln(R-\xi)-\frac{\eta}{q}\left(\tan^{-1}\frac{\xi\eta}{qR}+\tan^{-1}\frac{\xi}{q}\right)$	$\frac{1}{2}\left(\frac{\eta}{R-\eta}-\ln(R-\eta)\right)$
$\frac{1}{R(R+\eta)}$	$-\frac{1}{q}\tan^{-1}\frac{\xi\eta}{qR}$	$-\frac{1}{R+\eta}$
$\frac{1}{R(R-\eta)}$	$\frac{1}{q}\tan^{-1}\frac{\xi\eta}{qR}$	$\frac{1}{R-\eta}$
$\frac{1}{R(R+\eta)^2}$		$-\frac{1}{2(R+\eta)^2}$
$\frac{1}{R(R-\eta)^2}$		$\frac{1}{2(R-\eta)^2}$
$\xi\frac{1}{R(R+\tilde{d})^2}$	$-\frac{1}{R+\tilde{d}}$	
$\xi\frac{2R+\tilde{d}}{R^3(R+\tilde{d})^2}$	$-\frac{1}{R(R+\tilde{d})}$	
$\xi\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}$	$-\frac{1}{R(R+\tilde{d})^2}$	
$\frac{1}{R(R+\tilde{d})}-\xi^2\frac{2R+\tilde{d}}{R^3(R+\tilde{d})^2}$	$\frac{\xi}{R(R+\tilde{d})}$	
$\frac{1}{R(R+\tilde{d})^2}-\xi^2\frac{3R+\tilde{d}}{R^3(R+\tilde{d})^3}$	$\frac{\xi}{R(R+\tilde{d})^2}$	
$\frac{\eta}{R(R+\tilde{y})}+\frac{\cos\delta}{R+\tilde{d}}$		$\ln(R+\tilde{y})$
$\frac{\eta}{R(R+\tilde{d})}+\frac{\sin\delta}{R+\tilde{d}}$		$\ln(R+\tilde{d})$
$\frac{\eta}{R(R+\tilde{d})^2}+\frac{\sin\delta}{(R+\tilde{d})^2}$		$-\frac{1}{R+\tilde{d}}$
$\frac{\cos\delta}{R+\tilde{d}}-\frac{\eta\tilde{y}}{R(R+\tilde{d})^2}-\frac{\tilde{y}\cos\delta}{(R+\tilde{d})^2}$		$\frac{\tilde{y}}{R+\tilde{d}}$
$\frac{\sin\delta}{R+\tilde{d}}-\frac{\eta\tilde{d}}{R(R+\tilde{d})^2}-\frac{\tilde{d}\sin\delta}{(R+\tilde{d})^2}$		$\frac{\tilde{d}}{R+\tilde{d}}$

$$X_{11}=\frac{1}{R(R+\xi)}\qquad X_{32}=\frac{2R+\xi}{R^3(R+\xi)^2}\qquad X_{53}=\frac{8R^2+9R\xi+3\xi^2}{R^5(R+\xi)^3}$$
$$Y_{11}=\frac{1}{R(R+\eta)}\qquad Y_{32}=\frac{2R+\eta}{R^3(R+\eta)^2}\qquad Y_{53}=\frac{8R^2+9R\eta+3\eta^2}{R^5(R+\eta)^3}$$

(∗) Derivation of $\int\frac{d\eta}{R(R+\xi)}$

Since $\frac{1}{R(R+\xi)}=\frac{1}{R^2-\xi^2}-\frac{\xi}{R(R^2-\xi^2)}$ and $\left.\frac{1}{R^2-\xi^2}\right|_{\xi=\xi_1}^{\xi=\xi_2}=0$,

$$\int\frac{d\eta}{R(R+\xi)}=-\xi\int\frac{d\eta}{R(R^2-\xi^2)}=-\xi\int\frac{d\eta}{(\eta^2+q^2)\sqrt{\eta^2+q^2+\xi^2}}=\begin{cases}-\frac{1}{q}\tan^{-1}\frac{\xi\eta}{qR} & \text{for } q\neq 0\\ \frac{R}{\xi\eta} & \text{for } q=0\end{cases}$$

Here, we have used the mathematical formula $\int\frac{dx}{(x^2+a^2)\sqrt{x^2+a^2+b^2}}=\begin{cases}\frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\frac{x}{\sqrt{x^2+a^2+b^2}}\right) & \text{for } a\neq 0, b\neq 0\\ -\frac{\sqrt{x^2+b^2}}{b^2x} & \text{for } a=0, b\neq 0\end{cases}$